The Basics of Simple Interest (Time & Money)

Loans are a common part of business and life. Money is borrowed or lent for a period of time and interest is charged on the loan. **Interest** can be thought of as “the rent” charged for using the money. The formula to calculate simple interest is:

Interest = Principal × Rate ×Time or I = Prt

Rate (r) is the annual rate of interest, Time (t) is the interest period in years, and Principal (P) is the original amount of money borrowed or loaned.

We can compute the **future value** of a sum of money (or **maturity value**) by adding the original principal to the interest due. (Think of this as the value of a loan with interest that must be repaid.)

Future Value = Principal + Interest
S = P + I = P + Prt (substitute in the first equation above for interest)
S = P (1 + rt)

We can also rearrange this equation to solve for the principal value of money when the future or maturity value is known:

P = S/(1 + rt)

When given any 3 of the 4 variables (time, interest rate, principle or maturity value), we can solve for the one unknown.

Things become slightly more complicated when the terms of repayment of a loan are renegotiated to a new payment schedule. The value of money changes as time passes. (Think about how much more an income of $40,000 was worth 40 years ago than it is today!) The value of the original investment or payment on any particular day is a **dated value** (or equivalent value).

Since the value of money changes with time, <u>we cannot compare sums of money that are given at different points in time</u>. In order to compare money at different points in time, we choose a **focal date**, and move all the amounts of scheduled payments (or money) to that specific date. This lets us compare the money values without inflation effects.

The steps to solving a problem with money at different points in time are:
1. Draw a timeline and enter the focal date and the old and new payment schedules.
2. Move all money amounts to the focal date using the correct formula.
3. Once the amounts are all at the focal date, we can add, subtract, and compare monetary amounts.

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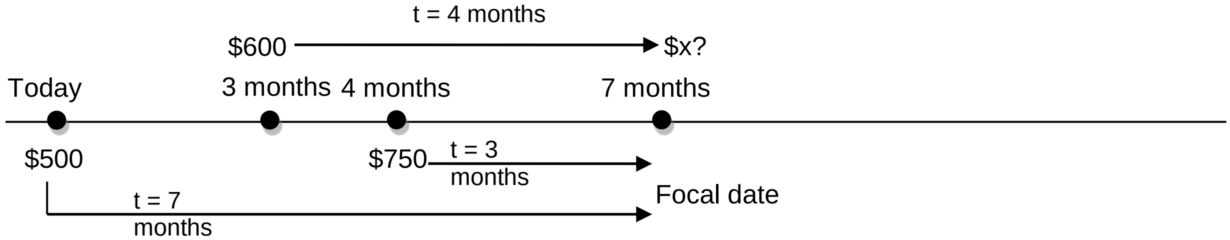
There are two directions in which we can move money towards a focal point in time: forwards or backwards.

Example 1: Moving money forwards in time

You were originally scheduled to make payments of \$500 due today and \$750 in four months from now. Instead you will settle the payment by paying \$600 in three months and a final payment in seven months from today. Determine the amount of the final payment at 7% p.a., using seven months from now as the focal point.

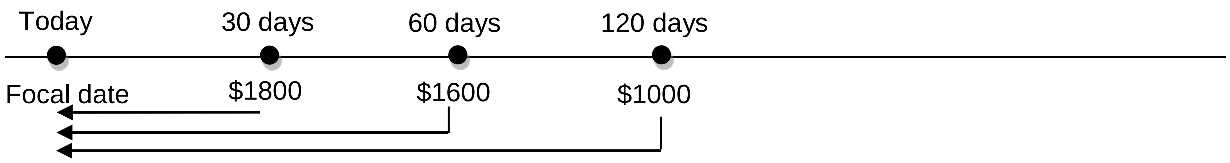


Solving this problem will require setting the originally scheduled payments equal to the replacement payments and determining \$x, the final payment. Since the focal point occurs in the future, we use the future value formula for each payment to bring it forwards in time to the focal date. Note that \$x does not need adjustment since this payment occurs at the focal point.

$$\$500[1+0.07(7/12)] + \$750[1+0.07(3/12)] = \$600[1+0.07(4/12)] + \$x$$
$$\$520.42 + \$763.13 = \$614 + x$$
$$x = \$669.55$$

<u>Example 2</u>: Moving money backwards in time

A debt is to be repaid by payments of \$1800 thirty days from now, \$1600 sixty days from now and \$1000 in 120 days from now. Determine the single payment that would fully repay the debt today. Simple interest allowed is 5.75%.



Now that we are moving money backwards to the focal point, we use the present value formula, $P = S/(1 + rt)$. Use this formula for each payment and sum them up to determine the single payment today.

$$= 1800/[1+0.0575(30/365))] + 1600/[1+0.0575(60/365))] + 1000/[1+0.0575(120/365)]$$
$$= 1791.58 + 1584.94 + 981.45$$
$$= \$4357.97$$

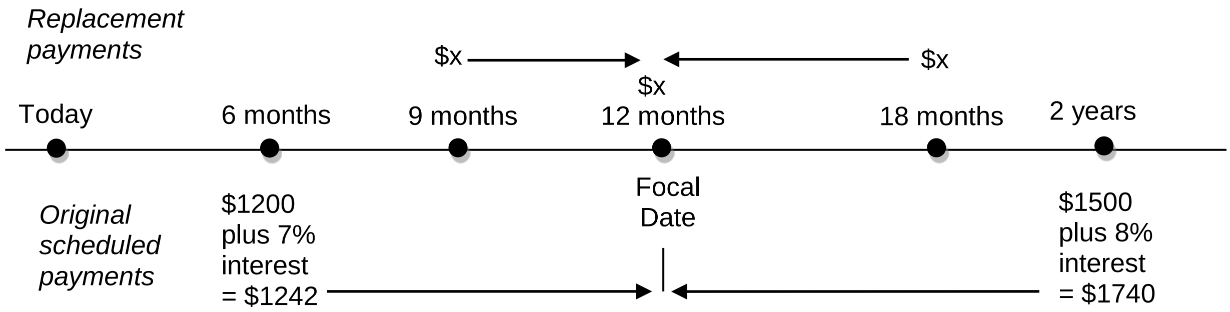
<u>Example 3:</u> Equivalent payments.

We have a debt of \$3000 that was to be paid today. Instead we reschedule to repay it in three equal payments: one made 30 days from now, one 60 days from now, and one 120 days from now. The focal date is today and interest is at 5.75%. What is the equal payment to be made?

Let x represent the value of the equal payment to be made on each of the three dates. We know the value of the loan on the focal date is $3000. The sum of our new three payments, once moved back to the focal date, must be equal. Since the payments occur AFTER the focal date, they represent a future value. We need to find the principal value at the focal date.

$3000 = x/[1+0.0575(30/365))] + x/[1+0.0575(60/365))] + x/[1+0.0575(120/365)]$
$3000 = 0.9953x + 0.9906x + 0.9814x$
$3000 = 2.9674x$
$x = \$1010.99$

Example 4: Maturity value and equivalent payment
This example will be a bit more difficult as it requires you to determine the maturity value of the original debts before proceeding. Let's say you have two debts, one of $1200 due in 6 months with interest at 7%, and $1500 due in 2 years with interest at 8%. These debts are to be repaid by making three equal payments due in 9, 12, and 18 months. What is the size of the equal payments if money is worth 12% and the chosen focal date is 1 year from today?



First, determine the maturity value of the original debts that are interest bearing.

(a) For the $1200 debt, we have 6 months worth of interest accumulated:

Maturity value, $S = \$1200[1+0.07(6/12)] = \$1242$

(b) For the $1500 debt, we have 2 years worth of interest :

Maturity value, $S = \$1500[1+0.08(2)] = \$1740$

Second, determine the dated value of the original payments at the focal point using 12%.

(a) Since the $1242 was due 6 months before the focal point, we move the money forward to find the maturity value:

$S = \$1242 [1+ 0.12(6/12)] = \$1316.52$

(b) Because the maturity value of $1740 is due 1 year after the focal point, we move the money backwards in time to find the present value:

$P = \$1740 /[1+ 0.12(1)] = \$1553.57$

Third, sum up the total of the old schedule of payments at the focal date:

$\$1316.52 + \$1553.57 = \$2870.09$

Fourth, move all the replacement payments to the focal date using the rate money is worth.

(a) Since the first payment is made 3 months before the focal date, we move that payment forward and find the maturity value:

$S = x(1+0.12(3/12)) = 1.0300x$

(b) Since the second payment is made on the focal date, it doesn't have to move anywhere in time: its value is $x

(c) The third payment is made 6 months after the focal point so we move the payment backwards in time to find the present value at the focal date:

$$P = x/(1+0.12(6/12)) = 0.9434x$$

Last, set the old schedule equal to the new schedule of payments and solve for x:

$$2870.09 = 1.0300x + x + 0.9434x$$
$$2870.09 = 2.97346x$$
$$x = \$965.26$$

Practice Problems

1. You were expecting debt payments of $600 due 5 months ago and $400 in 3 months from now. What single payment would you accept today for full repayment with interest allowed at 10% p.a. if the focal point is today?
2. Bailey has a textbook loan that was supposed to be paid in two payments of $1200 due 64 days ago and $1080 due in 22 days from now. What single payment would Bailey need to make 90 days from now to pay off the debts, if interest is to be 13.21% and the agreed upon focal date is 90 days from now?
3. Fred was originally scheduled to repay a debt by making two payments - $300 due 1 month ago, and $800 due in 4 months. Instead, he renegotiates to pay $600 today and the balance in 7 months. How much will the balance be if the focal point is today and the interest allowed is 5% p.a.?
4. Figaro scheduled debt payments to RBC of $1012 due 5 months ago and $1380 due today. Instead, he now offers to repay RBC $725 in 4 months and the balance in 8 months. If interest allowed is 6.25% and the focal date is in 8 months, what is the amount of his final payment?
5. Joanna took out a loan and had a payment schedule of $1600 due 5 months ago, and $1400 due in 8 months. She has rescheduled payments of the loan to be three equal payments due in 7 months, 9 months and 12 months. What is the size of the payments with the focal point at today and interest of 11%?
6. Cheesybites Co. owes two debts, one of $5000 due three months ago and the other of $4000 due in 8 months with interest at 10.5%. These debts are to be repaid by paying $3000 today and making two equal payments in 6 months and 12 months. What is the size of these payments if money is worth 12% and the focal date is today?
7. Xtreme Sports Inc. was supposed to make a payment of $1500 to their advertising agency one year ago. They are also scheduled to pay $1200 due with interest of 9% in 8 months. The advertising agency has agreed to accept three equal payments due today, 5 months from today, and a year from today at 10%. Find the amount of equal payments if the focal date is a year from today.

Solutions

1. $1015.24
2. $2,373.46
3. $502.38
4. $1777.92
5. $1077.15
6. $3329.04
7. $986.09