Solving Algebraic Equations & Word Problems

Every equation has a left side, an equal sign, and a right side. The equal sign tells us that the value of the left side is the same as the right side. (Imagine a set of perfectly balanced scales.) Algebra uses equations to describe relationships between variables and numbers; it allows us to solve for unknown variables. Below is an equation:

$$5a - 11 = 13 + 3a$$

To solve an equation for a variable (find its value), the basic plan is to isolate the variable on one side of the equation and make the coefficient next to the variable equal to 1. We do this by adding, subtracting, multiplying, or dividing until we isolate the variable. The key thing to remember is that **anything you do to the one side of the equation must be done to the other side.** If you don't do the same math operation to both sides, you are making the equation untrue (no longer equal).

Example: Solve the equation above.
Solution:

(1) Combine like terms. Subtracting 3a from both sides brings all the variables to one side where they can be combined.

$$5a - 3a - 11 = 13 + 3a - 3a$$
$$2a - 11 = 13$$

(2) Next, add 11 to both sides to combine the number terms and isolate the variable.

$$2a - 11 + 11 = 13 + 11$$
$$2a = 24$$

(3) Last, divide both sides by 2 to bring the coefficient of "a" to 1.

$$2a \div 2 = 24 \div 2$$
$$a = 12$$

An algebraic expression may also require simplification before solving. For instance, if there are fractions in the expression, multiply each term of the equation by the lowest common denominator to make solving the equation easier.

Example: Solve $\frac{4}{5}x - \frac{3}{2} = \frac{7}{10} + \frac{1}{4}x$
Solution:

(1) Since this equation involves fractions, the first step is to find the lowest common denominator and multiply every term by it. LCD = 20

$$20\left(\frac{4}{5}x\right) - 20\left(\frac{3}{2}\right) = 20\left(\frac{7}{10}\right) + 20\left(\frac{1}{4}x\right)$$
$$16x - 30 = 14 + 5x$$

(2) Subtract 5x from both sides to bring all the x's to the same side and combine:

$$16x - 30 - 5x = 14 + 5x - 5x$$

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$$11x - 30 = 14$$

(3) Isolate the variable by adding 30 to both sides. Then divide by 11 to bring the coefficient of the variable to 1.

$$11x - 30 + 30 = 14 + 30$$
$$11x = 44$$
$$\frac{11x}{11} = \frac{44}{11}$$
$$x = 4$$

The same rules still apply when there is more than one variable in a problem (say, A and C) and you are asked to solve for one variable (A) in terms of another variable (in this case, C). Just remember to treat the variable you are NOT interested in solving for like a number.

Example: Solve the following equation for A:

$$0 = \frac{3}{2}(A + 4) - C$$

Solution:

(1) We want to isolate A on one side. To do this, we do BEDMAS backwards: everything *outside* the brackets first, addition and subtraction before multiplication and division. The first step is adding C to both sides.

$$0 + C = \frac{3}{2}(A + 4) - C + C$$
$$C = \frac{3}{2}(A + 4)$$

(2) To get to A, we can either multiply $^{3}/_{2}$ through the terms in the bracket and then isolate A, or we can save ourselves some work by multiplying each side by the reciprocal fraction, $^{2}/_{3}$, to remove the coefficient in front of the brackets.

$$C \times \frac{2}{3} = \frac{2}{3} \times \frac{3}{2}(A + 4)$$
$$\frac{2C}{3} = A + 4$$

(3) Finish isolating A by subtracting 4 from each side.

$$\frac{2C}{3} - 4 = A + 4 - 4$$
$$\frac{2C - 12}{3} = A$$

Practice Problems

Solve each of the following equations.

1. 13x = 26
2. −8x = 32
3. $-\frac{4}{3}x = -24$
4. $x + 12 = 20$
5. 2x + 15 = 33
6. 5y + 7 = 12y − 3
7. 4s + 9 = 13s − 11 − 5s
8. 4(3x + 7) = 2x + 58
9. 6 − 8(2x − 4) = −58
10. 4(5z − 9) − 3(2z + 8) = 108
11. $x + \frac{1}{5}x = 36$
12. $z + \frac{4}{3}z = \frac{7}{2}$
13. $-\frac{1}{6}x + \frac{5}{7} = \frac{1}{21} + \frac{1}{2}x$
14. $\frac{3}{8}x + 4 = \frac{221}{24} - \frac{2}{3}x$
15. $\frac{5}{6}(4x - 3) - \frac{2}{5}(3x - 4) = 6x - \frac{16}{15}(1 + 3x)$

Solve each of the following equations for the indicated variable.

(a) $F = \frac{kq_1q_2}{d^2}$ for d

(b) A = 2πr for r

(c) S = P(1 + rt) for P

(d) S = P(1 + rt) for r

(e) $F = \frac{mv^2}{r}$ for m

(f) $C = (F - 32)\frac{5}{9}$ for F

Solutions

1. x = 2 2. x = −4 3. x = 18 4. x = 8 5. x = 9 6. y = $^{10}/_{7}$
7. s = 5 8. x = 3 9. x = 6 10. z = 12 11. x = 30 12. z = $^{3}/_{2}$
13. x = 1 14. x = 5 15. x = $^{1}/_{4}$

(a) $d = \sqrt{\frac{kq_1q_2}{F}}$ (b) $r = \frac{A}{2\pi}$ (c) $P = \frac{S}{1+rt}$ (d) $r = \frac{S}{Pt} - \frac{1}{t}$ (e) $m = \frac{Fr}{v^2}$

(f) $F = \frac{9}{5}C + 32$

SOLVING WORD PROBLEMS

When you first read a word problem, the amount of information can be overwhelming. Read it a couple of times and break it down into the following pieces to make it easier: What are you supposed to find? What facts are given? What facts are implied? Then use a systematic approach to write the equation(s) describing the relationships between quantities in the problem. The following steps should be useful.

Step 1: Define the variable for the unknown quantity. Be clear about what the variable means (write it down)! For instance, saying “x is the hamburgers and y is the hot dogs” is too general. Is it how many of each there are? Their cost? The number of calories in each? Be specific.

Step 2: Identify any vocabulary words whose definition might be important in setting up equations like net sales, gross sales, dividends, etc.
Step 3: Translate the words in the problem into one or more equations. A good rule is that each sentence will be at least one equation. Remember – you need as many equations as you have unknowns to be able to solve for them all — if there are two unknowns, you'll need two equations.
Step 4: Solve your equations. For two- or three-variable problems, this means substituting one (or more) equations into another equation. Once you find one variable, use it to find the remaining variable(s). Make sure the number you found is what was asked for.

The table below lists some common words associated with math operations.

Keywords and what they generally mean in word problems				
Addition	**Subtraction**	**Multiplication**	**Division**	**Equality**
The sum of The total of Increased by More/more than Added to Exceeds Expands Greater than Gain/profit Longer	Less than Decreased by Subtracted from Difference between Diminished by Take away Reduced by Less/minus Loss Lower	Times Multiplied by [Fraction] of [Percent] of The product of Twice Double Triple Half	Divide(s) Divided by Divided into Half of Third Per Ratio	Equals Is/was/are Is equal to The result is What is left What remains The same as Gives/giving Makes Leaves

NOTES:
1. The word *and* is usually used as a conjunction: "The product of 4 *and* c" = 4c. It is also commonly used to represent addition (as in "two *and* two are four").
2. "15 more than a number" may be written "15 + n" or "n + 15".
3. "15 less than a number" is "15 − n", **not** "n − 15". "
4. To treat an expression as a single number, use parentheses: "Twice the sum of b and c" is "2(b + c)".

Example: This month's price per barrel of oil has increased by one-twentieth of last month's price to $88.56. Find last month's price per barrel of oil.
Solution:
(1) Unknowns: Let "p" represent last month's price per oil barrel.
(2) Translate: This month's price has increased by $(^1/_{20})$p. The price for this month would be written as p + $(^1/_{20})$p, or p(1 + $^1/_{20}$)
(3) Set up the equation: we have the algebraic expression for this month's price on one side and the number value of this month's price on the other side.
p + $^1/_{20}$(p) = $88.56
(4) Solving the equation, we find: p = $84.34
Check the answer: $84.34(1 + $^1/_{20}$) = $88.56 is true

Example: Sharim invested a total of $36,000 in two mutual funds. Her investment in the Equity Fund is $8,000 more than three times her investment in the Bond Fund. How much does Sharim invest in the Equity Fund?

Solution:

(1) Unknown: Let the amount invested in the Bond Fund be B. The amount invested in the Equity Fund is E.

(2) Translate. Two unknowns mean we should have two equations to solve.

First equation B + E = $36,000

Second equation E = 8000 + 3B

(3) Solve the equation by substituting the second equation into the first

B + $8,000 + 3B = $36,000

4B = $28,000

B = $7,000

Therefore, investment in Equity Fund, E = 3($7,000) + $8,000 = $29,000

Check: Total invested = $7,000 + $29,000 = $36,000 = total invested (correct)

Answer: Sharim invested $29,000 in the Equity Fund.

Practice Problems

1. The Vancouver Junior Hockey League spent $2,364 on hockey pucks. Lower-quality practice pucks cost $18 each, while high quality game pucks cost $26 each. If the League purchased 110 pucks in total, how many pucks of each type did they buy?
2. A. Vespucci bought two shirts online. A shipping fee of $5 per shirt was charged. HST of 12% was also charged on the shirt price and the shipping fee. If the total cost for two shirts was $113.12, what was the price per shirt?
3. Kate and Margo agree to open a bakery together. The partnership's capital is $65,000. If the partnership agreement stipulates that Kate invest $1,600 less than four-fifths of what Margo invests, how much did they each invest?
4. Max Planck spent $2,703 on different grade light bulbs for his experiment. He bought nine more fifteen-dollar bulbs than two times the number of ten-dollar bulbs and twenty-eight fewer eighteen-dollar bulbs than four-thirds the number of ten-dollar bulbs. How many of each type did he buy?
5. After selling $3 tickets to an event, Charlie Brown has $116 in his cash box, made up of quarters, loonies ($1 coins), and toonies ($2 coins). How many quarters does he have if the number of toonies is four more than one-third the number of loonies, and the number of quarters is sixteen fewer than four times the number of loonies?

Solutions

1. Let L represent low quality pucks. High quality pucks, H = 110 − L. The total amount spent on pucks is \$18L + \$26H = \$2364. Substitute the expression for H into the dollar value equation. Solve for L.
   L= 62 pucks, H = 48 pucks

2. \$113.12 ÷ 2 = \$56.56 (total cost for one shirt)
   Let P = price of one shirt
   \$56.56 = P + \$5 + 0.12(P + \$5); P = \$45.50/shirt

3. Let K be the amount Kate invests and M the amount Margo invests.
   K + M = \$65,000
   K = $\frac{4}{5}$M – \$1600
   Substitute the expression for K into the total investment equation and solve for M.
   Margo invests \$37,000; Kate invests \$28,000.

4. Unknowns: t = quantity of \$10 bulbs, f = quantity of \$15 bulbs, e = quantity of \$18 bulbs
   \$10t + \$15f + \$18e = \$2703
   f = 2t + 9
   e = $\frac{4}{3}$t − 28
   Substitute the expressions for f and e into the total cost equation and solve for t, the quantity of \$10 bulbs. Plug in the value for t into the equations for f and e, and find those values.
   Answer: t = 48 ; f = 105 ; e = 36

5. Let Q = number of quarters, L = number of loonies, T = number of toonies.
   \$116 = \$0.25Q + \$1L + \$2T (or 11600 = 25Q + 100L + 200T)
   T = $\frac{1}{3}$ L + 4
   Q = 4L – 16
   Take the expression for T and Q and substitute into the dollar value equation and solve for L. Coin quantities: 152 quarters, 42 loonies, 18 toonies.