



Ratio, Proportion & Percentage

RATIO

A **ratio** is a comparison of the relative values of numbers or quantities. We can write a ratio for any statement containing such a comparison. For example, if oranges cost \$10/kg, apples \$8/kg and pineapples \$20/kg, the ratio of fruit prices is written as 10:8:20.

Ratios can be written (or expressed) in any of the following ways:

- with "to", as in "7 to 2"
- with a colon, "7 : 2"
- as a fraction, " $\frac{7}{2}$ "
- as a decimal, "3.50"
- as a percent, "350%"

The numbers in a ratio are called the **terms of the ratio**.

Some guidelines about ratios:

- If more than two numbers or quantities are being compared, write the ratio using a colon.
- Don't include units of measurement. BUT, the terms of the ratio need to be in the same unit of measurement before writing the ratio. (e.g. 30 minutes to 2 hours becomes 30 min to 120 min or 30:120)
- When rates are expressed as ratios, the units of measurement are dropped even though the terms of the ratio represent different things, like time and distance. (e.g. 75 km/hr becomes 75:1)
- Ratios should not contain decimals when using "to" or a colon. Multiply all terms by 100 (or some other factor) to get rid of the decimal.
- If you have a ratio of fractions, multiply by the lowest common denominator to simplify.
- Ratios should be reduced to lowest terms by dividing by a common factor.

Ratios can also be used to solve word problems about allocation, or dividing a whole into a certain number of parts. The key to these problems is finding the whole, and then the parts.

Example: If three business partners invested a total of \$86,000 in a ratio of 5:3:2, how much did each partner invest?

Solution:

Method 1: The whole investment consists of $5 + 3 + 2 = 10$ parts. Each part of the investment is worth $\$86,000/10 = \$8,600$.

Partner 1 invests 5 of the 10 parts	$5 \times \$8,600 = \$43,000$
Partner 2 invests 3 of the 10 parts	$3 \times \$8,600 = \$25,800$
Partner 3 invests 2 of the 10 parts	$2 \times \$8,600 = \$17,200$

Method 2: There are 10 parts and each partner invests a fraction of the whole amount.

Partner 1 invests $\frac{5}{10}$ of \$86,000 =	$\frac{5}{10} \times \$86,000 = \$43,000$
Partner 2 invests $\frac{3}{10}$ of \$86,000 =	$\frac{3}{10} \times \$86,000 = \$25,800$
Partner 3 invests $\frac{2}{10}$ of \$86,000 =	$\frac{2}{10} \times \$86,000 = \$17,200$



Practice Problems

A. Simplify each of the following ratios.

1. 12 to 48 2. 81:27 3. 4 to 24 to 64 4. 21:42:14

B. Set up a ratio for each of the following and reduce to lowest terms. Eliminate decimals and fractions by using equivalent ratios.

5. 13 dimes to 8 nickels 11. $\frac{1}{2}$ to $\frac{3}{5}$
6. 10 days to 4 weeks 12. $\frac{5}{8}$ to $\frac{3}{4}$ to $\frac{4}{3}$
7. \$36 per day for 8 employees for 12 days
8. 500 metres in 6 minutes 13. $2\frac{2}{3}$ to $3\frac{1}{5}$
9. 1.25 to 8
10. 0.8 to 2.3 to 4.1

14. A library employs 6 librarians, 4 technicians, and 12 library aides. What is the ratio of librarians to technicians to library aides?
15. For each latte sold, the cost is made up of \$2.75 labour cost, \$1.20 food cost, and \$0.40 overhead. What is the ratio that exists between the three elements of cost?

C. Solve each of the following allocation problems.

16. Insurance cost for a building is distributed among renters based on their occupied square footage. If renter one occupies 200 sq ft, renter two 500 sq ft and renter three 300 sq ft and the total insurance is \$7,600, how should the insurance cost be distributed?
17. A consulting firm allocates its revenues between administration, marketing, and labour in the ratio $\frac{5}{18}$ to $\frac{2}{9}$ to $\frac{1}{2}$. If the total revenue from a project was \$12,450, how should it be distributed between the departments? Round answers to two decimals.
18. The return on an investment is allocated based on three partners' equity contributions to the cost of renovations of their restaurant. Marley, Fozziwick, and Sanders contributed \$64,000; \$82,000 and \$70,000 respectively to the renovations. If the net return on investment is \$399,600, how much does each partner receive?

Solutions

- A. 1. 1:4 2. 3:1 3. 1:6:16 4. 3:6:2
B. 5. 130:40 → 13:4
6. 10:28 → 5:14
7. 36:8:12 → 9:2:3
8. 500:6 → 250:3
9. 125:800 → 5:32
10. 8:23:41
11. 5:6 (multiply by 10)
12. 15:18:32 (multiply by 24)
13. 40:48 → 5:6 (convert to improper fractions first, then multiply by 15)
14. 6:4:12 → 3:2:6
15. 275:120:40 → 55:24:8
C. 16. renter 1 pays $\frac{2}{10}$ (\$7,600) = \$1,520; renter 2 pays $\frac{5}{10}$ (\$7,600) = \$3,800;
renter 3 pays $\frac{3}{10}$ (\$7,600) = \$2,280
17. Ratio: 5:4:9 18 pieces in total. Admin: \$3,458.33 Marketing: \$2,766.67
Labour: \$6,225
18. Marley \$118,400 ; Fozziwick \$151,700; Sanders \$129,500



PROPORTION

When two ratios are equal, they form a **proportion**:

$$\frac{a}{b} = \frac{c}{d} \quad \text{or} \quad a : b = c : d$$

In any proportion, the cross-products are equal: $a \times d = b \times c$

We can use this rule to solve equations where one term is unknown or in a word problem involving two ratios.

Example: Solve: $\frac{5}{25} = \frac{2}{x}$

Solution: Set up the cross products: $5 \cdot x = 25 \cdot 2$

Simplify: $5x = 50$

Solve for x by dividing both sides by the coefficient of x:

$$5x \div 5 = 50 \div 5$$

$$x = 10$$

Example: If it costs \$1.50 to cut 4 keys, how much would it cost to cut 18 keys?

Solution: Form a proportion for the problem. The units should match on each side of the equal sign (numerators should have the same units and so should the denominators):

$$\frac{\$1.50}{4 \text{ keys}} = \frac{x}{18 \text{ keys}}$$

Set up the cross-products:

$$1.50 \cdot 18 = 4 \cdot x$$

Solve for x:

$$(1.50 \cdot 18) \div 4 = 4x \div 4$$

$$6.75 = x$$

Answer: It would cost \$6.75 to cut 18 keys.

Practice Problems

A. Solve for the unknown.

1. $3 : n = 5 : 20$

2. $a : 42 = 5 : 6$

3. $50 : 75 = x : \frac{1}{2}$

4. $7 : 3.5 = 21 : z$

5. $0.6 : g = 0.78 : 0.325$

6. $\frac{7}{8} : \frac{3}{16} = t : \frac{5}{8}$

B. Use proportions to solve the following problems.

- 1) A box of 500 vitamin C tablets sells for \$4.80. How much would 125 tablets cost?
- 2) If you can open 12 cans of soup in 5 minutes, how many cans of soup could you open in 45 minutes?
- 3) A manufacturing plant can make 750 microwave ovens in 9 days. How large an order for microwave ovens can the plant fill in 33 days?
- 4) R. Habsburg has a three-fifths interest in a salt mining operation. He sold five-sevenths of his interest for \$4,500. (a) What was the total amount of Habsburg's interest in the operation before selling? (b) What is the total value of the salt mining operation?
- 5) In last fiscal quarter, Travel Inc.'s gross profit was four-sevenths of net sales, and their net income was two-fifths of gross profit. If net income was \$6800, find the gross profit and net sales for the last fiscal quarter.



Solutions

- A. (1) 12 (2) 35 (3) $\frac{1}{3}$ (4) 10.5 (5) 0.25 (6) $\frac{35}{12}$
B. (1) \$1.20 (2) 108 cans of soup (3) 2750 microwave ovens
(4) (a) \$6,300 (b) \$10,500 (5) Gross profit = \$17,000; Net sales = \$29,750

PERCENT

Recall that multiplying a number by a percent gives a percentage: 10% of 60 = $0.1 \times 60 = 6$

10% is the percent, also called the *rate*
60 is original number, also called the *base*
6 is the new number or *percentage*

$$\begin{aligned}\text{Percentage} &= \text{Rate} \times \text{Base} \\ \text{New number} &= \text{Percent} \times \text{Original Number}\end{aligned}$$

Both of these equations are expressing the same relationship, and one that you are already familiar with.

Example: If the actual sales of \$42,600 this month were 80% of the budgeted sales, how much was the sales budget for this month?

Solution: Let the sales budget be \$ x (the base). Since the actual sales (the percentage) were 80% (rate) of budgeted sales,

$$\begin{aligned} \$42,600 &= 0.80 \cdot x \\ \frac{\$42,600}{0.80} &= x \\ x &= \$53,250 \end{aligned}$$

Problems involving Increase or Decrease:

Percent Change

Change is often expressed in terms of a percent difference between two amounts. Phrases like: "is 10% *more than*", "is 35% *less than*", "is *increased by* 80%", or "is *decreased by* 45%" indicate a change. To solve *increase* or *decrease* problems, use the equation below:

$$\text{Original Number} \pm \text{Increase/Decrease} = \text{New Number}$$

where the increase/decrease (change) is expressed as a percent of the original number.

Example: How much is \$20 increased by 50%?

Solution: The increase is 50% of \$20 and the original number is \$20.

Let the new amount be x .

$$20 + 50\% \text{ of } 20 = x \rightarrow 20 + 10 = x$$

$$x = 30. \quad \text{Answer: The amount is } \$30.$$

Example: 90 is 50% less than what number?

Solution: 90 is the number after the decrease; the number before the decrease is unknown.

Let the original number be x , the decrease is 50% of x .

$$x - 50\% \text{ of } x = 90$$

$$x - 0.5x = 90 \rightarrow 0.5x = 90 \rightarrow x = 90 \div 0.5$$

$$x = 180 \quad \text{Answer: The original number is } 180.$$



Rate of increase or decrease

The phrase, “y is what percent more/less than x?”, indicates a problem of rate increase or decrease. In this type of problem the difference between the original number and the new number is expressed as a percent of the original number.

$$\text{Rate of change} = \frac{\text{Amount of change}}{\text{Original Number}} \times 100$$

Example: What percent less than \$200 is \$75?

Solution: The amount before the decrease is \$200.

The decrease is: $200 - 75 = \$125$.

The rate of decrease = $(125 \div 200) \times 100 = \frac{5}{8} \times 100 = 62.5\%$

Practice Problems

1. A house in Kitsilano was sold for 300% of what the owners originally paid. If the owners sold the property for \$1.2 million, how much did they originally pay for the house?
2. If C. Columbus has \$31.30 deducted from his biweekly pay check for EI, and the rate for EI is 3.90% of gross wages, what is the total amount of his pay check?
3. Alice Wong contributed \$36 towards a birthday present that cost a total of \$80. What percent of the total did she contribute?
4. \$475 is what percent more than \$125?
5. \$530 is what percent less than \$643.95?
6. In Quebec, a motorcycle sold for \$28,461.38 including 5% GST and 7.5% PST. How much did the motorcycle cost before tax?
7. A bartender whose salary was \$320 per week was given a raise of \$40 per week. What percent increase did the bartender receive?
8. A credit card carries a 15.2% interest rate. If the amount of interest accrued for last month was \$360, what is the balance on the card?
9. Revenue increased from \$1,062 last year to \$1,947 this year. What was the percent increase in profit?
10. Net income in November increased 18 $\frac{1}{2}\%$ over October's net income. If net income for October was \$25680, what was net income for November?
11. The CPI at the end of 2010 was 15% lower than at the end 2000. What was the CPI at the end of 2010, if the CPI at the end of 2000 was 120?
12. The earnings per share (EPS) of a company last year was \$1.12. This year the earnings per share is \$1.40. What is the percent change in EPS? What is the new EPS as a percent of the old EPS?

Solutions

- (1) \$400,000 (2) \$802.56 (3) 45% (4) 280% more (5) 17.7% less (6) \$25,299.00 (7) 12.5%
(8) \$2,368.42 (9) 83 $\frac{1}{3}\%$ (10) \$30,430.80 (11) 102 (12) 25%; 125%

Currency Conversions

Proportions can also be used to convert from one currency to another currency (or one unit to another unit). The exchange rate (given rate) always forms one side of the proportion equation and the second ratio contains the unknown value.



Example: What is \$200 Canadian dollars (C\$) in US dollars if C\$1 is worth US\$0.8969?

Solution: Set up the proportion and solve: $\frac{C\$200}{US\$x} = \frac{C\$1}{US\$0.8969}$ $x = (200)(0.8969) = \$179.38$

Index Numbers

An **index number** is the comparison of two dollar amounts at different points in time. The ratio of the two amounts is expressed as a percent, the percent symbol is dropped, and this is called an index number. The **base period** is the year to which all other periods' dollar amounts are compared. The index number of the base period is always 100. The difference between a period's index number and 100 is the relative percent change that has taken place.

Example: In 2004, the price of aluminum was \$1,400 per tonne. In 2008 the price was \$2,200 per tonne. Find the index numbers, and the relative change between 2004 and 2008.

Solution:

The earlier year will be the base period. The index number for 2004 is 100. The index number for 2008 is (rounded): $\frac{\$2,200}{\$1,400} = 1.57 \times 100 = 157\%$ The trend index number is 157 for 2008.

The relative percent change is $157 - 100 = 57\%$ increase.

The **Consumer Price Index (CPI)** is a trend number used to indicate changes in the overall price level of goods and services. The base period is 1992. The CPI can be used to determine the purchasing power of the dollar with the following equation:

$$\text{Purchasing power of the dollar} = \frac{\$1}{CPI} \times 100$$

The CPI can also be used to adjust nominal income (income in current dollars) to real income (income in base-period dollars). This allows for comparison of dollar amounts by correcting for inflation.

$$\text{Real Income} = \frac{\text{Income in Current Dollars}}{CPI} \times 100$$

Practice Problems

1. If the price of gasoline is C\$1.14 per litre, find the cost in US dollars per gallon. One US dollar is worth C\$1.05 and one US gallon is equal to 3.78 litres.
2. Mario & Luigi's plumbing business earned \$128,000 in 2000. If the Consumer Price Index in 2000 was 113.5 and in 2010 was 124.6, what did they have to earn in 2010 to keep up with inflation?
3. The Consumer Price Index for 2005 was 117.4 and for 2009 was 122.6. (a) Determine the purchasing power of the dollar in 2005 and 2009 (round to 2 decimal places). (b) Determine the index number for 2009 relative to 2005
4. Annabelle's annual incomes for 1995, 2000, and 2005 were \$41,000, \$46,000, and \$50,000 respectively. Given that the CPI for the three years was 101.3, 108.5, and 117.4, respectively, compute her real income for 1995, 2000, and 2005.

Solutions

1. US\$4.10/gal
2. \$140,518.06
3. (a) \$0.85 for 2005, \$0.82 for 2009 (b) 104.4
4. real income for 1995 = \$40,473.84; 2000 = \$42,396.31; 2005 = \$42,589.44

