Linear Systems of Equations

Any pair of linear equations (with two variables) can be solved by using algebra or graphing. To solve systems of equations algebraically, we can either use the **elimination** or **substitution method**. The strategy is the same for both methods: create one equation with one unknown.

METHOD 1: ELIMINATION

Step 1: The equations need to be in the same format. If they are not, pick one equation and rearrange it to match the other equation.

Example: $3x + 5y = 12$
$2y = 10 + 4x$

Solution: In the first equation, the x and y terms are on the left of the equal sign and the number is on the right. To make the second equation match, subtract 4x from both sides.

$$3x + 5y = 12$$
$$-4x + 2y = 10$$

Step 2: Pick one of the variables to eliminate. If there is a variable with the same coefficient in both equations, choose that one. If not, multiply either one or both equations by a factor so that the variable you picked to eliminate has the same coefficient in both equations.

Solution: In the example above, neither x nor y has the same coefficient in both equations. Let's choose to eliminate "x". The lowest common multiple of 3 and 4 is 12. Multiply each equation by the appropriate factor to give x a coefficient of 12.

$$4(3x + 5y = 12) \quad \rightarrow \quad 12x + 20y = 48$$
$$3(-4x + 2y = 10) \quad \rightarrow \quad -12x + 6y = 30$$

Extra tip: If an equation contains decimals or fractions, multiply by a factor that will eliminate the decimals or fractions. This will make it simpler to solve the system.

Step 3: Add the two equations if the coefficients have opposite signs; subtract the two equations if the coefficients have the same sign. This should eliminate the variable you chose and give you one equation with one kind of variable.

Solution: Since the coefficients of x have opposite signs, we should add the two equations. Add the left sides of both equations and the right sides of both equations:

$$12x - 12x + 20y + 6y = 48 + 30$$
$$26y = 78$$

Step 4: Solve the equation and plug the value for your variable back into one of the original equations to solve for the other variable.

Solution: Solving for y: $26y \div 26 = 78 \div 26$
$$y = 3$$

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Now plug into either the first or second equation and solve for x:

$$3x + 5(3) = 12$$
$$3x = 12 - 15 = -3$$
$$3x \div 3 = -3 \div 3$$
$$x = -1$$

METHOD 2: SUBSTITUTION

To solve a system using substitution, use the following steps:

Step 1: Pick one of the equations and rewrite the equation to isolate the variable of your choice (be smart about your selection and make it easier for yourself).

Step 2: Substitute the expression for the variable in step 1 into the other equation.

Step 3: Solve for the one variable and plug back into the original equation to find the other unknown.

Hint: This method should be used when you can easily write one variable in terms of the other. You've probably already solved word problems using this method!

Example: The Orpheum collected \$7,900 from the sale of 440 tickets. If the tickets were sold for \$15 and \$20 respectively, how many tickets were sold at each price?

Solution: Let f = the number of \$15 tickets and t = the number of \$20 tickets.

$$f + t = 440 \qquad (1)$$

$$\$15f + \$20t = \$7{,}900 \qquad (2)$$

Step 1: Solve equation #1 for f (isolate f on one side of the equation). This gives:

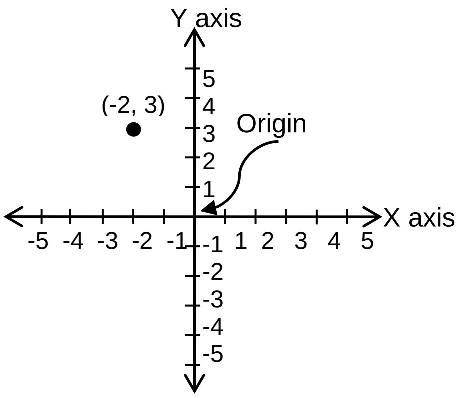
$$f = 440 - t \qquad (3)$$

Step 2: Substitute equation #3 into equation #2 for f. Then solve for t (step (c)).

$$\$15(440 - t) + \$20t = \$7{,}900$$
$$\$6{,}600 - \$15t + \$20t = \$7{,}900$$
$$\$6{,}600 + \$5t = \$7{,}900$$
$$\$5t = \$1300$$
$$t = 260$$

Step 3: Plug the value for t into equation 3 and solve for f.

METHOD 3: GRAPHING



The vertical axis is the y-axis, and the horizontal axis is the x-axis. The origin is at the point (0, 0) where the two axes intersect. The position of any point on a graph is given by an ordered pair of numbers (x, y) where the first coordinate gives the position on the x-axis and the second coordinate gives the position on the y-axis.

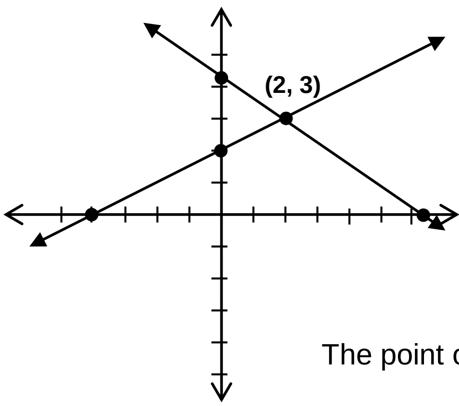
The point (−2, 3) is shown on the graph. To plot this point we go left 2 (since it is negative) from the origin on the x-axis and then up 3 on the y-axis.

To plot a line on a graph, use the following steps:
Step 1: Build a table of values of at least two ordered pairs for each equation.
Step 2: Plot the points on a graph.
Step 3: Draw a line through each pair of points.

Example: Solve by graphing:

$$2x + 3y = 13$$
$$x - 2y = -4$$

Solution: Create a table of values for each equation and graph a line from its equation. The two easiest points to plot are usually the x-intercept (where a line crosses the x-axis, y coordinate is 0) and the y-intercept (where a line crosses the y-axis, x coordinate is 0). This means for the x-intercept, we write 0 into the table, and then plug 0 into the equation and solve for y. We write the corresponding value for y below its x coordinate. For the y intercept, we write 0 for y in the table and plug 0 for y into the equation. Once we solve for x, we write that value in the column where y is 0. We can also choose a third point. Wherever you get the same ordered pair for x and y in BOTH equations is where the two lines intersect.



2x + 3y = 13			
x	0	6.5	2
y	4.33	0	3

x − 2y = −4			
x	0	−4	2
y	2	0	3

The point of intersection is (2, 3) so the solution is x = 2, y = 3.

Alternately we can use the slope-intercept form of the equations to solve for the intersection of two lines. Note that this intersection is the SAME solution you would get by using algebraic methods.

When a line is in slope-intercept form it has the general equation of:

$$y = mx + b$$

"y" must be isolated on one side of the equation with a coefficient of 1. "m" is the coefficient of "x" and represents the slope, or steepness of a line. Slope is also called rise over run, or the vertical change relative to the horizontal change between two points on a line. "b" represents the y-intercept, which as mentioned above, is where the line crosses the y-axis (with an x coordinate of 0).

There are two special cases for slopes – horizontal lines and vertical lines. Horizontal lines have the form y = #, where # is any number. These lines have a slope of zero. Vertical lines have the form x = #, where # represents any number. These lines have an undefined slope (because the "run" of the line is zero and a zero in the denominator of a fraction gives an undefined value).

Once you have an equation in slope-intercept form, you have one known point (the y-intercept) and from that point you can use the slope to get to the next point on the line.

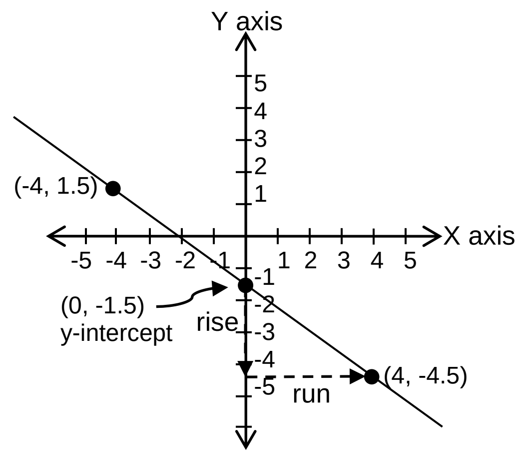
Example: Find the slope and y-intercept of 3x + 4y = −6. Graph the equation.
Solution: The first step is to get the equation in the form y = mx + b. So we solve the equation for y.

$$4y = -6 - 3x$$

$$y = -\frac{6}{4} - \frac{3}{4}x$$

$$y = -\frac{3}{2} - \frac{3}{4}x \quad \rightarrow \quad y = -\frac{3}{4}x - \frac{3}{2}$$

Now that we have the equation in the proper form, we can see that "b", the y-intercept is $-^{3}/_{2}$. The slope is $-^{3}/_{4}$. We can plot the y-intercept (0, $-^{3}/_{2}$) on the graph and then use the slope to find the next point by counting down 3 (rise) and right 4 (run), or alternately up 3 and left 4. Then draw a line through the points.



Notice that for a line with a negative slope, the line will tilt down towards the right. For a line with a positive slope, it should tilt up towards the right.

Practice Problems

A. Solve the following systems of equations algebraically.

1. $4x + 3y = 5$ $6x - 3y = 15$	4. $12z = 24 - 3y$ $40z - 4y + 4 = 0$	7. $\frac{x}{2} + \frac{3y}{5} = -\frac{11}{10}$ $\frac{5y}{12} + \frac{3x}{8} = -\frac{23}{24}$
2. $2x + 4y = 20$ $2x - 3y = -1$	5. $0.5r + 1.2s = 18.4$ $2.3r - 0.6s = 11.2$	
3. $5a + b = -10$ $4b = 5 - 5a$	6. $3.5x + 0.65y = 49.25$ $1.75x + 2.15y = 15.5$	8. $\frac{4x}{5} + \frac{3y}{4} = \frac{125}{10}$ $\frac{3x}{4} - \frac{2y}{3} = \frac{21}{6}$

B. Construct a table of values for each of the following equations.

9. y = 3x + 1
10. 4x = 2y + 8

C. Using algebra, find the slope and y-intercept of the lines represented by the equations below.

11. 5x – 6y = 12
12. x + $^{1}/_{4}$y = 2
13. 4y – 2 = 0
14. 5 + $^{1}/_{2}$x = 0

D. Solve the following systems of equations graphically.

15. $x + y = 5$ and $x - y = -5$
16. $x + y = 4$ and $x - y = -2$
17. $4x + 8y = 16$ and $x = 6$
18. $5y = 3x$ and $5y + 2x = 25$
19. $4y = 12x - 24$ and $2y + 4x = 3$

SOLUTIONS

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1) $x = 2, y = -1$
2) $x = 4, y = 3$
3) $a = -3, b = 5$
4) $y = 6, z = {}^1/_2$
5) $r = 8, s = 12$
6) $x = 15, y = -5$
7) $x = -7, y = 4$
8) $x = 10, y = 6$
9) Answers may vary

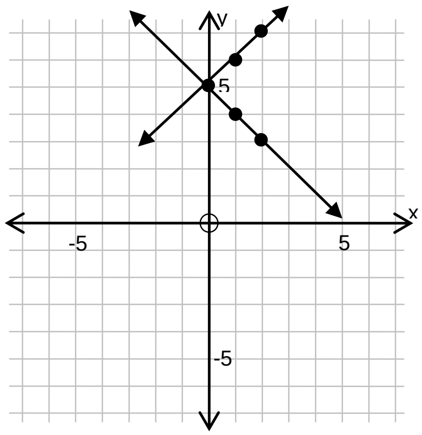
x	0	$-{}^1/_3$	1	2
y	1	0	4	7

10) Answers may vary

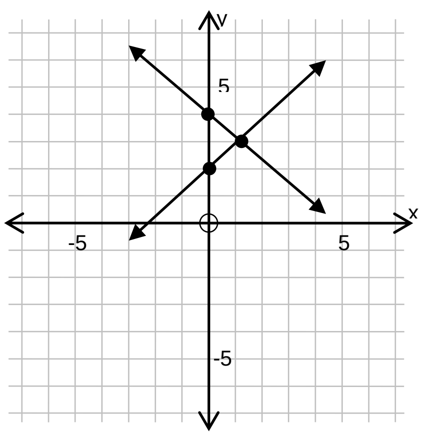
x	0	2	1	3
y	−4	0	−2	2

11) $y = \frac{5}{6}x - 2$ slope, $m = {}^5/_6$; y-intercept, $b = -2$
12) $y = -4x + 8$ slope, $m = -4$; y-intercept, $b = 8$
13) $y = \frac{1}{2}$ This is a horizontal line. slope, $m = 0$; y-intercept, $b = ½$
14) $x = -10$ This is a vertical line. slope, m is undefined; no y-intercept.

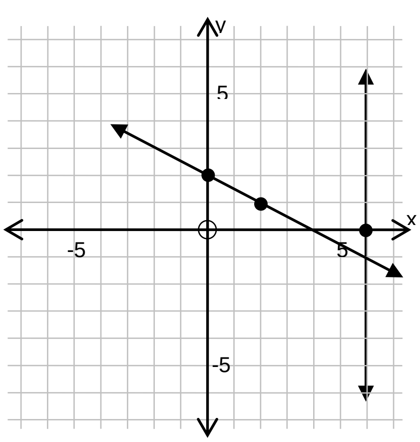
15) solution: x = 0, y = 5



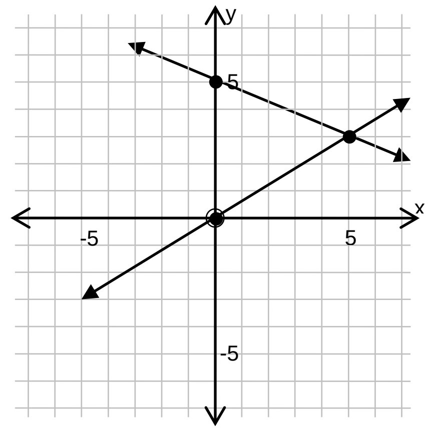
16) solution: x = 1, y = 3



17) solution: x = 6, y = −1



18) solution: x = 5, y = 3



19) solution: $x = {}^3/_2$, $y = -{}^3/_2$