



## Cost-Volume-Profit Analysis

The basic principles of CVP analysis were covered in business math. CVP analysis can be done both graphically, through plotting the equations for Total Revenues and Total Costs (Fixed Costs + Variable Costs), and algebraically, through solving the two linear equations of total revenues and total costs.

The **breakeven** point (whether in sales \$ or in sales units) represents the point where the sales revenue (TR) exactly covers the fixed costs and variable costs (TC). In other words, the breakeven point occurs where there is \$0 profit (and \$0 loss). On a graph, it is the point where the TR curve and TC curve intersect. Any sales above the breakeven point will result in an operating profit for the business. Any sales below the breakeven point will result in an operating loss.

The **contribution margin** is the difference between total sales revenue and total variable costs; this amount contributes to paying fixed costs and beyond that, providing profit. The **contribution margin unit (CMu)** is the amount of money per unit of good sold that contributes towards fixed costs and profit. It is the difference between the sales price per unit (SPu) and the variable cost per unit (VCu). The **contribution margin ratio** is the expression of the portion of each dollar of sales that goes towards fixed costs and profit (CMu/SPu).

Some basic equations that you might use for solving for the breakeven point are below:

Breakeven sales revenue in dollars (\$)

$$= \frac{\text{Fixed Costs}}{1 - \left(\frac{\text{Variable Costs}}{\text{Sales Revenue}}\right)} = \frac{\text{Fixed Costs}}{1 - \left(\frac{\text{Variable Cost per Unit}}{\text{Sales Price per Unit}}\right)} = \frac{\text{Fixed Costs}}{\text{Contribution margin ratio}}$$

$$\text{Breakeven sales units} = \frac{\text{Fixed Costs}}{\text{Sales Price Unit} - \text{Variable Costs Unit}} = \frac{\text{Fixed Costs}}{\text{Contribution margin per unit}}$$

The value for VC/SR and VC per unit/SP per unit is also called the **variable cost percent**. The contribution margin is equal to 1 – Variable Cost %.

*Example 1: If the fixed costs (FC) for a year of operation are \$45,000, the variable expenses (VC) are \$27,000 and sales revenue (SR) is \$112,000, what is the breakeven sales revenue (BESR)?*

*Solution:*

$$\text{BESR} = \frac{\text{Fixed Costs}}{1 - (\text{Variable Costs}/\text{Sales Revenue})} = \frac{45,000}{1 - (27,000/112,000)} = \frac{45,000}{100\% - 24.1\%} = \$59,289$$



Note in these types of problems, we calculate the variable cost percentage first and round that number (24.1%). So the contribution margin ratio is  $100\% - 24.1\% = 75.9\%$ , which is what we divide by to find the break even sales revenue. All dollar values should be rounded (no cents).

Another way to show CVP analysis is a **contribution margin income statement** where fixed costs, variable costs, the contribution margin and the operating income are shown. For example 1:

Contribution Margin Income Statement	
Sales revenue	\$112,000
Less: Variable Costs	<u>(27,000)</u>
Contribution margin	85,000
Less: Fixed Costs	<u>(45,000)</u>
Operating Income	40,000

If there is a required amount of operating income (OI) that a business wants to make, the BESR equation changes to express the *required sales* in terms of FC, OI, and contribution margin (CM) ratio.

$$\text{Required sales in \$ (or units)} = \frac{\text{FC} + \text{OI}}{\text{CM ratio (or CM per unit)}}$$

If there are any adjustments to fixed costs, this would also affect the required sales \$ for a certain OI level. Changes to fixed costs are added in the top of the equation to figure out how the sales revenue must change. The same is true if we are talking about an investment and we want to see a specific return on investment (this becomes a new OI).

*Example 2: A business operates with a variable cost percentage of 68%. The fixed costs of operation last year were \$37,000. The owner is giving her employees a raise which results in an increased fixed cost of \$9,200 this year. What is the required sales revenue in order to achieve an operating profit of \$16,000 assuming FC and the VC percentage are the same?*

*Solution:*

$$\text{Required SR} = \frac{\text{FC} + \text{new FC} + \text{OI}}{1 - \text{VC}\%} = \frac{\$37,000 + 9,200 + 16,000}{1 - 0.68} = \$194,375$$

In the case where a business sells differently priced goods (with differing variable costs), the contribution margin income statement is a bit more complicated, and you will need to calculate a **weighted-average contribution margin** per unit. You will be given a **sales mix**, which tells you the relative proportion in which a company's goods are sold.

To find the weighted average CM, first find the contribution margin per unit of each individual good. Then multiply each CM by the quantity of that good in the sales mix. Total the contribution margins and then divide by the total quantity of goods in the sales mix. This is the weighted average CM.



*Example 3: A company sells apples and oranges in a sales mix of 4 apples : 5 oranges. The selling price for apples is \$0.50 each and for oranges is \$0.70 each. The variable cost of apples is \$0.20 per unit and for oranges \$0.30 per unit. Find the weighted average contribution margin.*

*Solution: The contribution margin for apples is  $(0.50 - 0.20) \times 4 = \$1.20$ . The contribution margin for oranges is \$2.00; the total contribution margin is  $\$1.20 + \$2 = \$3.20$ . To find the weighted-average contribution margin per unit, take \$3.20 divided by the total of goods in the sales mix:  $\$3.20/(4 + 5) = \$3.20/9 = \$0.36$ .*

To find the number of units sold of each good at the breakeven point, take the total breakeven units (FC divided by the weighted-average contribution margin per unit) and multiply by the fraction each good represents of the sales mix.

If there are two departments in a business, you may be asked to find how changes in sales revenue to one or both departments impact the contribution margin and profit levels of the business. Use the following formula to determine the required sales based on each department's contribution margin and their percent of total revenue.

$$\text{Required Sales} = \frac{\text{FC} + \text{OI}}{(\text{Dept A Rev\%} \times \text{Dept A CM ratio}) + (\text{Dept B Rev\%} \times \text{Dept B CM ratio})}$$

*Example 4: A hotel has a room department and food department. The hotel's FC are \$350,000 a year. The room department's revenue is \$450,000 and the food department's revenue is \$300,000. The variable costs for the room department are \$225,000 and the food department's variable cost is \$102,000. What increase in sales revenue is needed to double the current operating income (a) by increasing only room sales revenue or (b) by increasing revenue jointly between the departments, assuming the revenue ratio stays the same?*

*Solution: First find the CM ratio for each department and each department's sales revenue % of the total sales revenue. Find the current operating income.*

Room revenue % =  $\$450,000/(\$450,000 + \$300,000) \times 100 = 60\%$

Food revenue% = 40%

Room Dept CM ratio =  $1 - (225,000/450,000) = 1 - 50\% = 50\%$

Food Dept CM ratio =  $1 - (80,000/250,000) = 1 - 34\% = 66\%$

Current Operating Income = TR – TC =  $\$700,000 - (\$350,000 + \$327,000) = \$23,000$

(a) To double the income would be to increase it by 23,000. If the increase needs to come from room revenues, then we just use the CM ratio for rooms:  $23,000 \times 0.50 = \$11,500$  increase in revenue is needed.

(b) To double the income jointly from both departments' revenue,

$$= \frac{23,000}{(60\% \times 0.5) + (40\% \times 0.66)} = \frac{23,000}{30\% + 26.4\%} = \frac{23,000}{56.4\%} = \$40,780$$

The increase in revenue needed is \$40,780.



## Practice Problems

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- Fixed costs are \$142,500 and the variable cost percentage is 35%. What is the breakeven sales revenue?
- A bar has fixed costs of \$178,000 and the contribution margin ratio is 56%. Find the breakeven sales revenue.
- If a business has fixed costs of \$215,000 and goods selling for \$12.50/unit with a variable cost of \$6.40/unit, find the breakeven sales units and revenue.
- A hotel has sales revenue of \$423,000 and variable costs averaging 48%. The average room cost is \$30/unit. Fixed costs are \$208,000. What is the breakeven sales revenue? What is the current operating income? If the hotel wants to increase its operating income by \$4,200, how much additional sales revenue is required? If variable costs increase to 54% and the hotel wants to maintain the increase in operating income of \$4,200, what is the required sales revenue?
- Company A has sales revenue of \$500,000 and fixed expenses of \$196,000. If the number of units sold is 16,000 and the contribution margin ratio is 0.44, find the variable expenses, operating income (loss), and contribution margin per unit.
- A coffee shop sells small, medium, and large coffees in a sales mix of 2:3:1. The variable costs per unit for small, medium, and large coffees are \$1.20, \$1.50, and \$1.90 respectively. The sales price per unit for small, medium, and large coffees are \$2.50, \$3.00, and \$3.50. The contribution margin income statement for the year is shown below.

Contribution Margin Income Statement	
Sales Revenue	84,000
Variable Expenses	(42,240)
Contribution Margin	41,760
Fixed Expenses	(36,000)
Operating Income	5,760

- Prepare a contribution margin income statement using the weighted-average technique for the three different coffee sizes.
- Find the breakeven sales in total units, and for each size of coffee.
- Find the breakeven sales in dollars.
- If the variable costs of coffee increase by 5% for all sizes and the shop wants to maintain an operating income of \$5,760, how many more units have to be sold to achieve break even?
- How many units of each coffee should have to be sold to achieve an



operating income of \$13,320 with the change in variable costs above?

7. Fill in the missing information in the table:

	<b>A</b>	<b>B</b>	<b>C</b>
Sales price per unit	\$30		\$25
Variable Cost per unit	\$12		
Fixed Cost			
CM per unit			\$13
CM ratio		45%	
Break even units	250		
Break even Sales			
Target Sales units	300	200	
Target Sales		\$4,000	\$6,950
CM			
Operating Income		\$840	(\$234)



## Solutions

1. \$219,230.77
2. \$317,857.14
3. 35,246 units (round to the nearest whole number); 35246 units × \$12.50/unit = \$440,575 is the BESR
4. BESR = \$400,000; current operating income = \$11,960; additional sales revenue = \$8,077; required sales revenue = \$487,304
5. Variable expenses = \$280,000; Operating income = \$24,000; Contribution margin per unit = \$13.75
6. (a)

	Small	Medium	Large	Total
Sale price per unit	\$2.50	\$3.00	\$3.50	
Less: Variable cost per unit	( 1.20)	( 1.50)	(1.90)	
Contribution margin per unit	1.30	1.50	1.60	
Sales mix in units	× 2	× 3	× 1	6
Contribution margin per unit	<u>2.60</u>	<u>4.50</u>	<u>1.60</u>	<u>8.70</u>
Weighted-average CM per unit				<u>1.45</u>

- (b) Breakeven sales units =  $\frac{\$36,000}{\$1.45/\text{unit}} = 24,828$  units  
 Breakeven sales of small coffees = 24,828 × (2/6) = 8,276 units  
 Breakeven sales of medium coffees = 24,828 × (3/6) = 12,414 units  
 Breakeven sales of large coffees = 24,828 × (1/6) = 4,138 units
- (c) Breakeven sales dollars =  $SP_s \times Q_s + SP_m \times Q_m + SP_L \times Q_L$   
 Breakeven sales dollars =  $\$2.50 \times 8,276 + \$3 \times 12,414 + \$3.50 \times 4,138$   
 = \$72,415
- (d) Change in VC means that small VC = \$1.26/unit, medium VC = \$1.58/unit, large VC = \$2.00/unit. The average CM per unit is now \$1.37.  
 Required sales units =  $(36,000 + 5,760) / \$1.37 = 30,482$  units  
 30,482 – 24,828 = 5,654 units more
- (e) Total units sold = 36,000; small coffees = 12,000; medium coffees = 18,000; large coffees = 6,000

7. Fill in the missing information in the table:

	A	B	C
Sales price per unit	\$30	\$20	\$25
Variable Cost per unit	\$12	\$11	\$12
Fixed Cost	\$4,500	\$960	\$3848
CM per unit	\$18	\$9	\$13
CM ratio	60%	45%	52%
Break even units	250	107	296
Break even Sales	\$7,500	\$2,140	\$7,400
Target Sales units	300	200	278
Target Sales	\$9,000	\$4,000	\$6,950
CM	\$5,400	\$1,800	\$3,614
Operating Income	\$900	\$840	(\$234)

