Function Notation

If you went to an accountant and asked how much you'd have to pay in taxes this year, the accountant would probably respond by asking, "How much did you make last year?" He can't answer your question until he has some numbers from you to start his calculation. Once he knows those numbers, he'll do the same math that he's done for many other clients to figure out how much you owe in taxes.

Functions are like this. For any particular function, you give it a number, it performs some calculations using that number, and it gives you a result. It doesn't matter what the number is at the start, the calculations will be the same.

One of the nice things about functions is we don't have to know what the calculated result is for us to talk about it. The tax accountant can talk to you about "the amount of taxes you'll have to pay on your income" even if he doesn't know the dollar value yet. He's describing the answer by telling you which set of calculations he's going to do (the ones on the tax form) and the number he's going to start with (your income).

In math, we like to label things with symbols and letters. In the same way that we can represent a number by using a variable like x, we name functions with letters and indicate what numbers they operate on. The symbol for a function usually looks like **$f(\ )$**. Inside the parentheses, we put the number or expression that we are going to apply the function to. (This is called the argument of the function.) When we're defining what calculations the function performs, we use a variable like x. The function that adds 3 to any number would look like this:

$$f(x) = x + 3$$

We can then refer to the result when we apply this calculation to any number or expression. If we want to write, "the answer we get when we add three to nine" we can write $f(9)$. We would read these symbols as "f of nine".

You might be wondering why we bother with this notation when we could easily write "9 + 3". As you progress through mathematics, you'll encounter functions that are more complicated than this one, where the easiest way to refer to a result is that it's the answer you get when you perform a particular calculations on a number. An example of this that you're already familiar with is fractions.

The answer to the problem 3 ÷ 7 is a repeating decimal. The length of the repeating part is six digits. To write this number down *exactly* it would be tedious to have to memorize those digits. We'd have to do that for all the other answers to division problems that don't work out evenly. Instead we refer to "the answer you get when you divide 3 by 7" as a fraction: $3/7$. Fraction form tells us what numbers we use in the calculation and what operation to apply to them, even if it doesn't convey its value. Fractions are a form of

Authored by Darren Rigby

function notation on two numbers. Change the numbers and you change the value.

Example 1: Given the function $f(x) = 5 - x$, find (a) $f(3)$ (b) $f(½)$ (c) $f(8)$ (d) $f(q)$ (e) $f(a + b)$.

Solution: To get the values, we can simply plug in and solve.

(a) $f(3) = 5 - [3] = 2$.
(b) $f(½) = 5 - [½] = 4½$.
(c) $f(8) = 5 - [8] = -3$.
(d) $f(q) = 5 - q$.
(e) $f(a + b) = 5 - (a + b) = 5 - a - b$.

That's how it is with function notation. Whatever the argument of the function is, plug it in. Some functions refer to their arguments more than once:

Example 2: Given the function $f(x) = |x + 3| + x^2$, find (a) $f(6)$ (b) $f(-7)$ (c) $f(0.23)$ (d) $f(z)$ (e) $f(x - 4)$.

Solution: We plug the argument of the function into the definition of the function everywhere we see x:

(a) $f(6) = |[6] + 3| + [6]^2 = |9| + 36 = 9 + 36 = 45$.
(b) $f(-7) = |[-7] + 3| + [-7]^2 = |-4| + 49 = 4 + 49 = 53$.
(c) $f(0.23) = |[0.23] + 3| + [0.23]^2 = |3.23| + 0.0529 = 3.2829$.
(d) $f(z) = |z + 3| + z^2$
(e) $f(x - 4) = |[x - 4] + 3| + [x - 4]^2 = |x - 1| + x^2 - 8x + 16$.

In (e), we saw the variable from the function's definition reused, but changed a bit. This is nothing to worry about. We plug it in just like anything else.

Example 3: Given the function $f(x) = 4x + 7$, solve (a) $f(x) = 11$ (b) $f(x) = -5$ (c) $f(x) = 0$.

Solution: This is a harder problem, and it's another reason to use function notation. It lets us refer to an x value that will give the result that we want. To solve these problems, we plug in a value for $f(x)$ in the definition equation and solve for x:

$f(x) = 4x + 7$

(a) $11 = 4x + 7 \rightarrow 11 - 7 = 4x + 7 - 7 \rightarrow 4 = 4x \rightarrow 4 \div 4 = 4x \div 4 \rightarrow 1 = x$
(b) $-5 = 4x + 7 \rightarrow -5 - 7 = 4x + 7 - 7 \rightarrow -12 = 4x \rightarrow -12 \div 4 = 4x \div 4 \rightarrow -3 = x$
(c) $0 = 4x + 7 \rightarrow 0 - 7 = 4x + 7 - 7 \rightarrow -7 = 4x \rightarrow -7 \div 4 = 4x \div 4 \rightarrow -\frac{7}{4} = x$.

We can also use the output of one function as the input of another.

Example 4: Given the functions $f(x) = -x + 5$ and $g(x) = x^2 - 9$, find (a) $f(g(2))$ and (b) $g(f(2))$.

Solution: Order of operations says that we work from the inside out, so we find the value of the function on the inside before we work on the one on the outside:

(a) $g(2) = [2]^2 - 9 = 4 - 9 = -5$; $f(g(2)) = f(\,[-5]\,) = -[-5] + 5 = 5 + 5 = 10$.
(b) $f(2) = -[2] + 5 = 3$; $g(f(2)) = g(\,[3]\,) = [3]^2 - 9 = 9 - 9 = 0$.

Notice that while we started with the same number, 2, in both cases, the answers weren't the same. The order in which the functions are applied makes a difference.

EXERCISES

A. Given the function $f(x) = x + 5$, find:

1) $f(6)$
2) $f(0)$
3) $f(25)$
4) $f(s)$
5) $f(a)$
6) $f(b - 3)$
7) $f(y - 5)$
8) $f(m^2 + n)$

B. Given the function $g(x) = 2x^3 - 4x$, find:

1) $g(-2)$
2) $g(0)$
3) $g(10)$
4) $g(t)$
5) $g(r)$
6) $g(x + h)$
7) $g(\frac{1}{2}k)$
8) $g(7) - g(3)$

C. Given the function $f(t) = |t - 4|$, find:

1) $f(7)$
2) $f(1)$
3) $f(0)$
4) $f(x)$
5) $f(z)$
6) $f(-t)$
7) $f(Ay + B)$
8) $f(j^2 + j)$

D. Given the function $f(x) = 3x - 1$, solve:

1) $f(x) = 8$
2) $f(x) = -4$
3) $f(x) = -13$
4) $f(x) = 0$
5) $f(x) = 30$
6) $f(x) = \frac{1}{2}$
7) $f(x) = 0.8$
8) $f(x) = 3p - 1$

E. Given the functions $g(x) = x^2 - 8$ and $h(x) = 4x + 1$, find:

1) $g(5)$
2) $h(3)$
3) $g(h(2))$
4) $h(g(-4))$
5) $g(g(3))$
6) $h(h(\frac{1}{5}))$
7) $g(h(x))$
8) $h(g(x))$

F. Write a function in function notation that expresses the following instructions in mathematical symbols:

1) Take a number, x, and multiply it by 5, then add 3 to the result.
2) Take a number, x, and add 3 to it, then multiply the result by 5.
3) Take a number, t, and square it, then add 4 times the number and 4 to the result.

4) Find the absolute value of 6 less than the square of a number, y.

5) Take a number, k, and subtract six from it, then cube the result.

G: In estimating how much lumber to buy (in units of board feet) for a basement renovation, a contractor takes the number of feet of wall space in the basement, w, multiplies by 2, and then adds 5 to account for waste.

1) Write the function, f, the contractor is using to estimate the board feet, b, he needs for a job. [*Hint: if you have trouble writing the formula, do parts (2) and (3) of this question and see what calculations you do to get the answer, then try part (1) again.*]

Determine how many boards the contractor would order for a job with:

2) 60 ft. of wall space

3) 28 ft. of wall space

4) 135 ft. of wall space

5) 320 ft. of wall space

How much wall space would there be in a job if the contractor orders:

6) 105 board feet (bd ft)

7) 277 bd ft

8) 63 bd ft

9) 413 bd ft

10) What function could you write that solves the sort of problem in (G6) through (G9)?

SOLUTIONS

A: (1) 11 (2) 5 (3) 30 (4) s + 5 (5) a + 5 (6) b + 2 (7) y (8) $m^2 + n + 5$

B: (1) −8 (2) 0 (3) 1960 (4) $2t^3 - 4t$ (5) $2r^3 - 4r$
(6) $2x^3 + 6x^2h + 6xh^2 - 4x + 2h^3 - 4h$ (7) $\frac{1}{4}k^3 - 2k$ (8) 658 − 42 = 616

C: (1) 3 (2) 3 (3) 4 (4) $|x - 4|$ (5) $|z - 4|$ (6) $|-t - 4|$ *or* $|t + 4|$ (7) $|Ay + B - 4|$
(8) $|j^2 + j - 4|$

D: (1) x = 3 (2) x = −1 (3) x = −4 (4) $x = \frac{1}{3}$ (5) $x = \frac{31}{3}$ (6) $x = \frac{1}{2}$ (7) x = 0.6
(8) x = p

E: (1) 17 (2) 13 (3) $g(9) = 73$ (4) $h(8) = 33$ (5) $g(1) = -7$ (6) $h(\frac{9}{5}) = \frac{41}{5}$
(7) $g(4x + 1) = 16x^2 + 8x - 7$ (8) $h(x^2 - 8) = 4x^2 - 31$

F: (1) $f(x) = 5x + 3$ (2) $f(x) = 5(x + 3) = 5x + 15$ (3) $f(t) = t^2 + 4t + 4$
(4) $f(y) = |y^2 - 6|$ (5) $f(k) = (k - 6)^3 = k^3 - 18k^2 + 108k - 216$
[It would not be necessary to expand $(k - 6)^3$ since it's much easier to calculate in its unexpanded form.]

G: (1) $f(w) = 2w + 5$ (2) 125 boards (3) 61 boards (4) 275 boards (5) 645 boards
(6) 50 ft. (7) 136 ft. (8) 29 ft. (9) 204 ft. (10) $f(b) = \frac{b - 5}{2}$