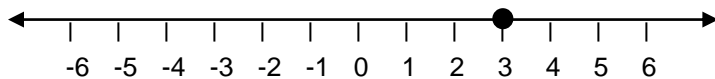
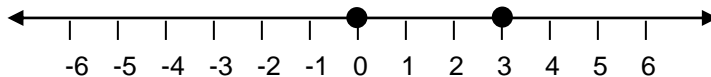


## Inequalities and the Number Line

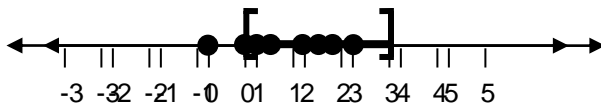
The number line is a useful tool for both understanding and expressing inequalities. Recall that the number line is a horizontal line with tick marks showing number values. Usually we only mark integers on the number line, but we can express any real number as a point on the line (whether it is on a tick mark or falling between the tick marks). The number line below has a closed circle at the 3 tick mark. Another way we might express this is using an algebraic equation, like  $x = 3$ . There is only one value for  $x$ .



When  $x$  is not a single value but a range of values, we can also use the number line to show multiple solutions for  $x$ . Let's say  $x$  is any number between 0 and 3, including 0 and 3. First, mark the minimum and maximum boundaries on the number line with closed circles for now.



To indicate that  $x$  can be any number between 0 and 3, we imagine that we put a point at every single value that is between 0 and 3... eventually it would get so crowded that it makes more sense to just draw a line between the two boundary values. Now that we know the range of values the inequality covers, we use the standard notation of square brackets to indicate the boundary values zero and three are included in the solution. The brackets face inwards, enclosing the range of  $x$  values.



To write this type of inequality algebraically, we use the form  $a \leq x \leq b$ , where  $a$  and  $b$  are any numbers and  $a$  is less than  $b$ . In this case, we would write  $0 \leq x \leq 3$ .

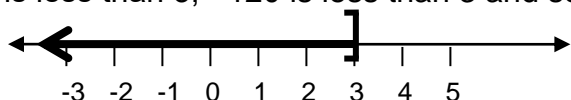
We could also have a case where  $x$  can be any number between 0 and 3, but not including 0 and 3. What would this look like? On the number line, a rounded bracket means that value is the "boundary", but it's not possible for  $x$  to be exactly equal to that value. We would redraw the graph from above with rounded brackets at the two ends of the range. The inequality expression now looks like:  $0 < x < 3$ .



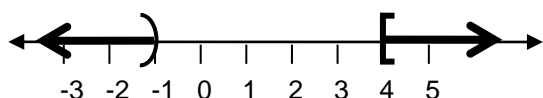
What about drawing a single-sided inequality on the number line, like  $x$  is less than or equal to 3? Since  $x$  can be equal to 3, we know a square bracket goes at 3. We know it points to the left because  $x$  can be any number less than 3. (If you're not sure which way it points, try putting a dot for every value where  $x$  is less than 3, then draw the bracket - the bracket should look like it's going to eat all the points).



Just like before, instead of drawing individual dots for all the solutions, we use a solid line to show all the values  $x$  can take. But what do we do at the end of the solid line of solutions? The number line itself has arrows to indicate that the values keep going on forever in the negative direction (to the left) and positive direction (to the right). Similarly, when we draw our line for the inequality, we have to add an arrow pointing in the negative direction because there is no boundary in that direction;  $-4$  is less than 3,  $-60$  is less than 3,  $-120$  is less than 3 and so on to negative infinity.



There's one other type of inequality you might encounter with a number line, called an "or" or "disjunction" inequality. For example,  $x$  is less than  $-1$  or  $x$  is greater than or equal to 4. If you see "or" in the statement of an inequality, it's easiest to approach each part separately. Let's look at the first statement,  $x$  is less than  $-1$  ( $x < -1$ ): we've already plotted something similar to this in the example above. Since it's a *less than* sign, we know we want to use a curved bracket and we want all the values that are less than  $-1$ . Remember with the number line, if you want *less than* a number, you go to the left of that number (also you could recall that the higher a negative number, the smaller it is). Now let's add the second statement in:  $x$  is greater than or equal to 4 ( $x \geq 4$ ). We use a closed bracket, pointing in the direction of values greater than four.



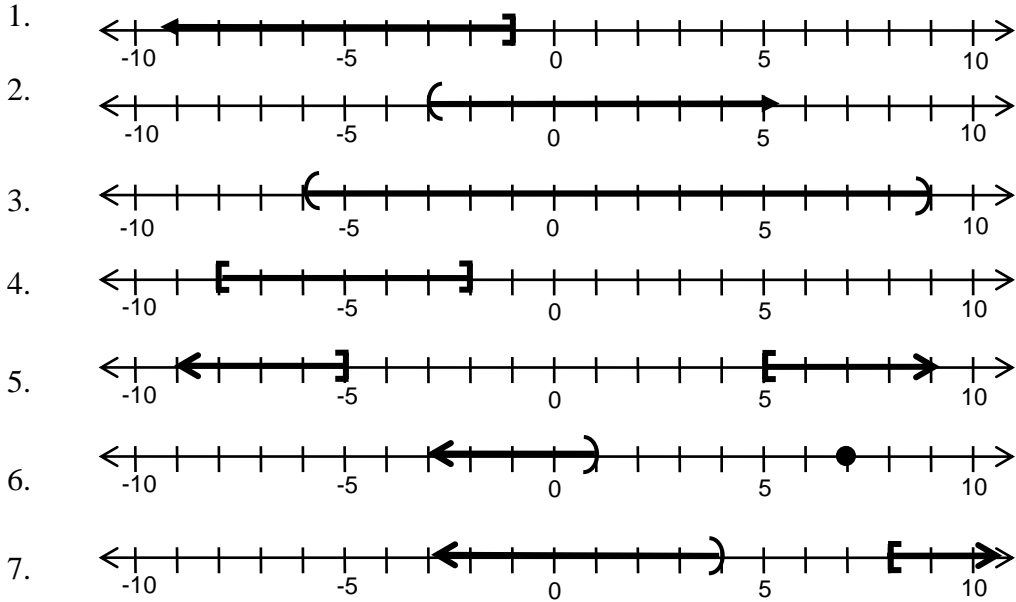
## EXERCISES

A. Express the following inequalities using a number line:

1.  $x = 5$
2.  $x \leq 0$
3.  $x \geq 2$
4.  $-1 \leq x \leq 2$
5.  $x = 2$  or  $x = -6$
6.  $x = -3$  or  $x > 1$
7.  $-3 < x < 5$
8.  $-4 \leq x < 2$
9.  $x < 0$  or  $x \geq 3$
10.  $x \leq 5$  or  $x > 6$

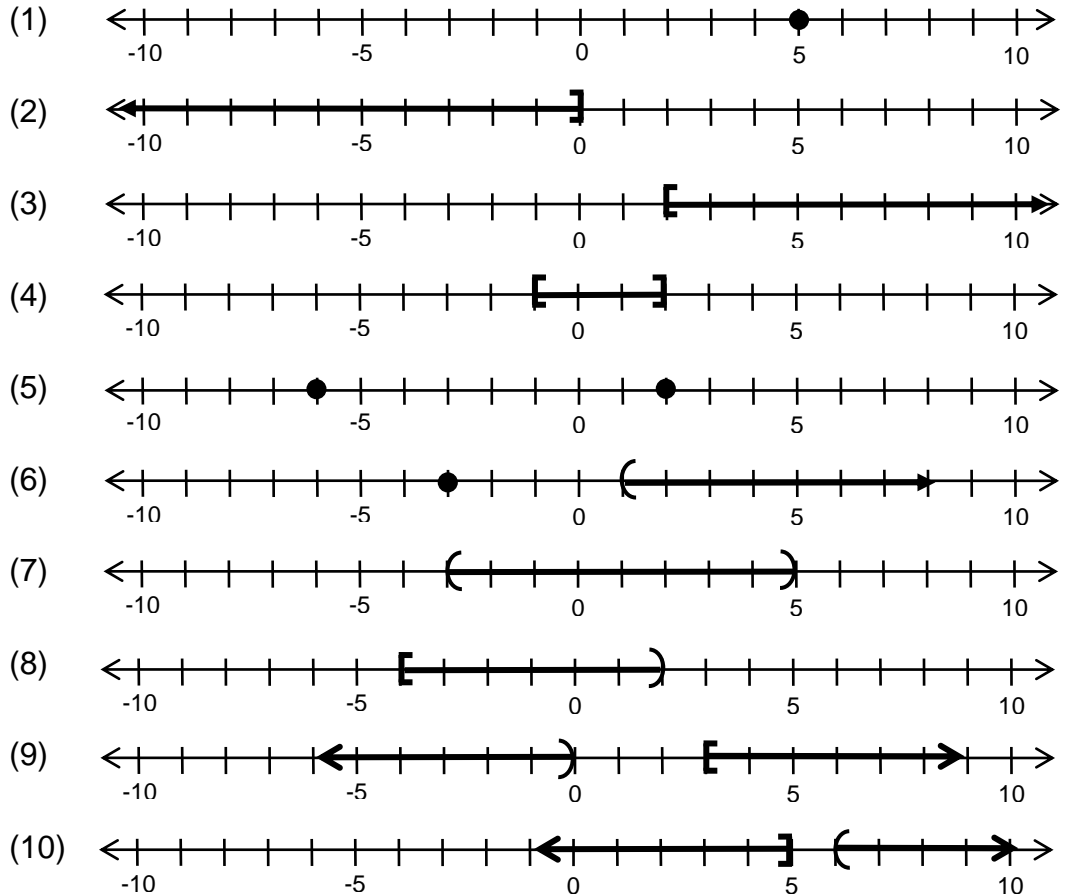


B. Given the following number lines, express the inequality



**SOLUTIONS**

A:



B. (1)  $x \leq -1$  (2)  $x > -3$  (3)  $-6 < x < 9$  (4)  $-8 \leq x \leq -2$  (5)  $x \leq -5$  or  $x \geq 5$   
 (6)  $x < 1$  or  $x = 7$  (7)  $x < 4$  or  $x \geq 8$

