Set Notation & Interval Notation

Many algebra problems have a single solution. If we have $x + 3 = 8$, the only answer is 5, and we can simply write $x = 5$. Other problems have multiple solutions or a range of solutions. There are two main ways to report answers to a question like this: interval notation and set notation.

SET NOTATION

Set notation is useful especially when we have a small, finite number of solutions, rather than a range of solutions. Take the equation $x^2 = 9$. There are two answers: 3 and −3. The list of all possible solutions to a problem is called its **solution set** and we should write it as a set using roster notation: $\{-3, 3\}$. The curly brackets (brace brackets) indicate that the answer is a list and that −3 and 3 are the only two acceptable answers. A solution in roster notation *can* have more than two numbers in the brackets.

It's also possible to write the solution set to a problem by describing the solutions rather than by listing all of them. If we were asked, "What quantities of money can be withdrawn from a typical ATM?" and the ATM only dispenses $20 bills, then the answers are 20, 40, 60, 80, and so on. We could write $\{20, 40, 60, \ldots\}$ as a way of listing the answers, or we could use **set-builder notation** to say how to calculate the answers: $\{x \mid x = 20k, k \in \mathbb{N}\}$. We read this as: The solutions are x, where x is 20 times k, and k is a natural number. This is a very precise answer, and more precise than your teacher is likely to ask for.

INTERVAL NOTATION

Interval notation is used whenever the answers to a problem form one or more continuous ranges of the number line. This frequently happens in inequalities.

Take for example, $x^2 < 9$. After some thought, it should be obvious that any number between −3 and 3 (but not including either number) is a solution to the problem. We express this in interval notation by enclosing the numbers that are the endpoints of the solution in brackets. We use **round brackets** or parentheses when the interval does not include the endpoints, and **square brackets** when the interval does include the endpoints. Here, since the solution interval doesn't include those numbers on the end, we write: $(-3, 3)$. If the question were $x^2 \le 9$, −3 and 3 would be valid solutions. We use square brackets to mark endpoints included in the solution: $[-3, 3]$. We can also use both bracket types in expressing a solution. For $4 < x \le 7$, the interval runs from 4 to 7, and 4 is not a solution, but 7 is. We write: $(4, 7]$.

Sometimes there's no endpoint. For the question $x \ge 12$, there's a lowest possible solution, but no highest possible solution. We use the infinity symbol to show a lack of an endpoint, and we must always use a round bracket with it; infinity isn't a number, so it can't be a solution. We can't include it as part of a solution set. We write $[12, \infty)$. We

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use $-\infty$ for solutions with no lowest endpoint: $x \le 12$ is expressed as $(-\infty, 12]$.Sometimes there are two intervals in the solution. For $x^2 \ge 9$, any number greater than 3 is a solution and so is any number less than −3, including 3 and −3. The solutions fall in two intervals that are separated by a gap. We indicate this by borrowing a symbol from set theory: the union symbol, ∪. This symbol is often translated as "or", meaning that any point in this interval *or* that interval is a valid solution. The solution to $x^2 \ge 9$ is written $(-\infty, -3] \cup [3, \infty)$. The solution to $x \ne -4$ would be $(-\infty, -4) \cup (-4, \infty)$, indicating that the "gap" between the two intervals is only as large as the number −4 itself.

If any number is a solution to an equation or inequality, as in $x^2 \ge 0$, then we write $\mathbb{R}$ in set notation ("all real numbers") or $(-\infty, \infty)$ in interval notation. If no number is a solution, as in $x^2 = -5$, then we write $\varnothing$ in either notation. This is the symbol for the null set, meaning the solution set is empty.

EXERCISES

A. Is 0 a solution to the problem whose solution sets are given below?

1) $[-2, 5]$
2) $(-2, 5]$
3) $\{-2, 5\}$
4) $(-7, 0]$
5) $\{x \mid x = k + 0.5, \text{ where k is an integer}\}$
6) $\{-15, -14, -13, \ldots 10, 11, 12\}$
7) $(-\infty, -5] \cup (-1, 6)$
8) $[-3, -1] \cup [8, 10]$

B. Write the solution sets to the following problems in the appropriate notation.

1) $x^2 < 25$
2) $|x| \le 8$
3) $x < 7$ and $x \ge 2$
4) List the factors of 12.
5) $x > 7$ and $x \le 2$
6) How much money can be withdrawn from ATMs that dispense $20 and $50 bills?
7) $x > 13$
8) $x < 7$ or $x \ge 2$
9) State the domain of $\frac{x+5}{x-4}$.
10) $x > 7$ or $x \le 2$

SOLUTIONS

A: (1) yes (2) yes (3) no (4) yes (5) no (6) yes (7) yes (8) no

B: (1) $(-5, 5)$ (2) $[-8, 8]$ (3) $[2, 7)$ (4) $\{1, 2, 3, 4, 6, 12\}$ (5) $\varnothing$ (6) $\{20, 40, 50, 60, 70\ldots\}$ or $\{x \mid x = 20a + 50b$, where a and b are whole numbers; a and b can't both be $0\}$ (7) $(13, \infty)$ (8) $(-\infty, \infty)$ or $\mathbb{R}$ (9) $(-\infty, 4) \cup (4, \infty)$ (10) $(-\infty, 2] \cup (7, \infty)$