Systems of Equations

There are three possible methods of solving systems of equations: **elimination**, **substitution**, and **graphing**. (Graphing, of course, only works on systems of two equations, unless you have three-dimensional paper.) In elimination and substitution, the strategy is the same: to remove variables from the system one by one until you have one equation and one unknown left.

METHOD 1: ELIMINATION

To solve a system using elimination:

a. Pick two equations from the system, and multiply one of the equations (or both equations) by a factor so that the coefficients on one of the variables will cancel when the two equations are added. (e.g. 6x and −6x)
b. Add the resulting equations to each other.
c. Repeat Steps a and b, always targeting the same variable, until you have one fewer equation than you started with. You must use each of your original equations at least once. If you do this, you will eliminate one variable.
d. Repeat Steps a, b and c until you have only one variable left. Solve.
e. Work backwards through the variables you eliminated to determine their values by plugging in values for the variables you know.

Example 1: Solve by elimination:

$2x + 3y = 13$ ①

$x - 2y = -4$ ②

Solution: $-2x + 4y = 8$ ③ ←Multiply equation ② by −2

Now we add equations #1 and #3, and get:

$0x + 7y = 21$ ④ ① + ③

$y = 3$

Now we plug our value for y into one of the equations we started with:

$2x + 3[3] = 13$

$2x + 9 = 13$

$2x = 4$

$x = 2$

Therefore the solution is x = 2, y = 3.

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METHOD 2: SUBSTITUTION

To solve a system using substitution:

a. Pick a variable and pick an equation. Rewrite the equation to isolate the variable.
b. Substitute the expression you got for the variable in Step a into all the other equations.
c. Repeat Steps a and b on each new batch of equations until you have one equation and one unknown left. Solve.
d. Work backwards through the variables you eliminated to determine their values by plugging in values for the variables you know.

Example 2: Solve by substitution:

$$2x + 3y = 13 \quad ①$$

$$x - 2y = -4 \quad ②$$

Solution: $x = -4 + 2y$ ③ ←Rearrange equation ② to solve for x

We substitute equation #3 into equation #1:

$$2[-4 + 2y] + 3y = 13 \quad ④ \qquad ③ \rightarrow ①$$

$$-8 + 4y + 3y = 13$$

$$7y = 21$$

$$y = 3$$

Now we plug our value for y into one of the equations we started with, as before.

In systems of equations, it doesn't matter which method you use, or which variable or equation you select at any point—the answer you get will be the same. Some choices may make the calculations easier on you, however.

It's possible that you may try one of these methods, and the result is an expression such as "0 = 0" or "0 = 2". In these cases, the system does not have one solution.

If you get a nonsense answer, like "1 = 3", then the system is **inconsistent**. There is no solution. There is no set of numbers that can satisfy all the equations at the same time.

If you get an obvious answer, like "5 = 5", then the *equations* are **dependent** (not the system!). There are infinite solutions.

Any system of equations that has a solution (meaning there is one solution or infinite solutions) is **consistent**. If there are *not* infinite solutions (meaning there is one solution or no solution), then the equations are **independent**.

There are three possible descriptions of a system then: inconsistent and independent (a set of parallel lines), consistent and dependent (the same line), or consistent and independent (a set of non-parallel lines that intersect once).

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METHOD 3: GRAPHING

To solve a system by graphing:

a. Graph each of the equations on the xy-plane.
b. The coordinates of the lines' point of intersection is the solution.
   Note: If the lines are parallel, the system is inconsistent, and it has no solution.
   If the lines overlap, the equations are dependent, and there are infinite solutions.

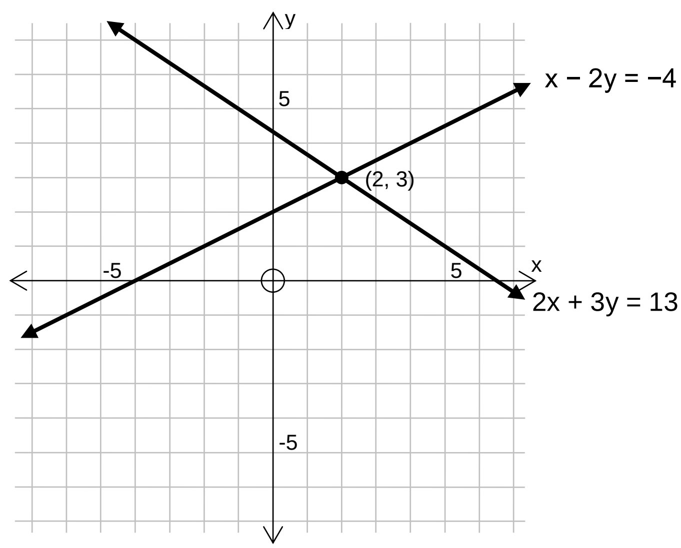
Example 3: Solve by graphing:

$$2x + 3y = 13 \quad ①$$

$$x - 2y = -4 \quad ②$$

Solution:

If you don't know how to graph a line from its equation, you can always create a table of values.



The point of intersection is (2, 3), so the solution is x = 2, y = 3.

EXERCISES

A. Solve the systems of equations. Use any of the three methods.

1) $2x + 2y = -8$
   $3x + y = 6$

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2) $7x - 4y = 15$
$4x + 7y = -10$

3) $2g - h = 17$
$-3g - 5h = 7$

4) $x - 3y = 5$
$2x - 5y = 12$

5) $x + y + z = 6$
$2x - y + z = 3$
$3x - y - z = -2$

6) $x + y = 5$
$y + z = 2$
$x + z = 1$

7) $3a + 4b + c = 8$
$2a - b - c = 0$
$a - 5b - 3c = -7$

SOLUTIONS

A. (1) $x = 5, y = -9$ (2) $x = 1, y = -2$ (3) $g = 6, h = -5$ (4) $x = 11, y = 2$
(5) $x = 1, y = 2, z = 3$ (6) $x = 2, y = 3, z = -1$ (7) $a = 1, b = 1, c = 1$

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