



Factoring

SPECIAL PRODUCTS

Perfect Squares

$$(a + b)^2 = a^2 + 2ab + b^2 ; (a - b)^2 = a^2 - 2ab + b^2$$

Difference of Squares

$$a^2 - b^2 = (a + b)(a - b)$$

Sum of Cubes

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

Difference of Cubes

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

EQUIVALENCES

Factoring Out -1

$$b - a = -(a - b)$$

Commutative Law of Addition

$$a + b = b + a$$

Reversing Differences

$$(b - a)^n = (a - b)^n \text{ when } n \text{ is even, and}$$

$$(b - a)^n = -(a - b)^n \text{ when } n \text{ is odd}$$

COMMON ERRORS

$$(a + b)^2 \neq a^2 + b^2 \quad (a - b)^2 \neq a^2 - b^2 \quad (a + b)^3 \neq a^3 + b^3 \quad (a - b)^3 \neq a^3 - b^3$$

$a^2 + b^2$ is not factorable. See "Special Products" above for the details of how to deal with these expressions.

EXERCISES

A. Factor out the greatest common factor:

1) $8t + 8$

6) $-7y^4 + 21y^2 - 7$

2) $5w - 35$

7) $r^2s - rs^2$

3) $-4h - 4k$

8) $2y(3y - 2) - (3y - 2)$

4) $\pi r^2 - 5\pi$

9) $3z^2(2z - 1) - 6(1 - 2z)$



5) $-x^2 - x$

10) $5(4 - y) - b(y - 4)$

B. Factor by grouping:

1) $ax + 3bx + ay + 3by$

6) $-5xyz + 4x^2z + 2x^2y - 10xz^2$

2) $2xy + cy - 6bx - 3bc$

7) $how + bow - hit - bit$

3) $2xy^2 - 8y^2 + x - 4$

8) $26qrv - 39rtuw - 56pqsv + 84psuw$

4) $b - 5z + 5 - bz$

9) $-28t^3 + 4t^9 - t^6 + 7$

5) $6x^2 + 18x - 8x - 24$

10) $3eh + en - 8at - 4nt - 12ht + 2ae$

C. Factor ($a = 1$):

1) $x^2 + 9x + 20$

6) $x^2 - 5x + 6$

2) $x^2 + 7x + 6$

7) $x^2 + 5x + 6$

3) $x^2 - 7x + 6$

8) $x^2 + x - 6$

4) $x^2 + 5x - 6$

9) $x^2 - x - 6$



5) $x^2 - 5x - 6$

10) $x^2 - 120 - 7x$

D. Factor ($a \neq 1$):

1) $6x^2 + 19x - 20$

6) $-x^2 + 3x - 2$

2) $6x^2 - 13x - 63$

7) $x^2y - 5xy - 36y$

3) $6x^2 + 17x + 7$

8) $6x^2 - 13x - 15$

4) $3x^2 + 24x - 60$

9) $12x^2 + 29x + 6$

5) $-8x^2 + 42x - 49$

10) $6x^2 - 12 - x$

E. Factor:

1) $x^2 - 49$

5) $x^3 - 27$

2) $x^2 - 8x + 16$

6) $m^6 - n^6$ (*Hint: Squares first.*)

3) $25x^2 + 60x + 36$

7) $m^6 + n^6$

4) $100x^2 + 140x + 49$

8) $x^3 + 125$



9) $3n - 48n^5$

15) $h^2 + 2hk + k^2$

10) $x^3 + y^6$

16) $a^4 - 13a^2 + 36$

11) $a^8 - b^8$

17) $h^4 - 12h^2 + 36$

12) $-128 + 2c^2$

18) $1 - y^6$

13) $64r^2 + 100s^4$

19) $(x + y)^2 - (x - y)^2$

14) $x^4z^4 - y^4z^4$

20) $(x + 1)^3 - 1$

SOLUTIONS

- A. (1) $8(t + 1)$ (2) $5(w - 7)$ (3) $-4(h + k)$ (4) $\pi(r^2 - 5)$ (5) $-x(x + 1)$ (6) $-7(y^4 - 3y^2 + 1)$
 (7) $rs(r - s)$ (8) $(3y - 2)(2y - 1)$ (9) $(2z - 1)(3z^2 + 6)$ (10) $-(y - 4)(b + 5)$
- B. (1) $(x + y)(a + 3b)$ (2) $(y - 3b)(2x + c)$ (3) $(2y^2 + 1)(x - 4)$ (4) $-(b + 5)(z - 1)$
 (5) $(6x - 8)(x + 3) = 2(3x - 4)(x + 3)$ (6) $x(2x - 5z)(y + 2z)$ (7) $(b + h)(ow - it)$
 (8) $(2qv - 3uw)(13rt - 28ps)$ (9) $(t^6 - 7)(4t^3 - 1)$ (10) $(2a + 3h + n)(e - 4t)$
- C. (1) $(x + 4)(x + 5)$ (2) $(x + 1)(x + 6)$ (3) $(x - 1)(x - 6)$ (4) $(x - 1)(x + 6)$
 (5) $(x - 6)(x + 1)$ (6) $(x - 3)(x - 2)$ (7) $(x + 2)(x + 3)$ (8) $(x - 2)(x + 3)$
 (9) $(x - 3)(x + 2)$ (10) $(x - 15)(x + 8)$
- D. (1) $(6x - 5)(x + 4)$ (2) $(3x + 7)(2x - 9)$ (3) $(2x + 1)(3x + 7)$ (4) $3(x + 10)(x - 2)$
 (5) $-(2x - 7)(4x - 7)$ (6) $-(x - 1)(x - 2)$ (7) $y(x - 9)(x + 4)$ (8) $(x - 3)(6x + 5)$
 (9) Not factorable. (10) $(3x + 4)(2x - 3)$
- E. (1) $(x + 7)(x - 7)$ (2) $(x - 4)^2$ (3) $(5x + 6)^2$ (4) $(10x + 7)^2$ (5) $(x - 3)(x^2 + 3x + 9)$
 (6) $(m - n)(m + n)(m^2 - mn + n^2)(m^2 + mn + n^2)$ (7) $(m^2 + n^2)(m^4 - m^2n^2 + n^4)$
 (8) $(x + 5)(x^2 - 5x + 25)$ (9) $-3n(2n - 1)(2n + 1)(4n^2 + 1)$ (10) $(x + y^2)(x^2 - xy^2 + y^4)$
 (11) $(a^4 + b^4)(a^2 + b^2)(a + b)(a - b)$ (12) $2(c + 8)(c - 8)$ (13) $4(16r^2 + 25s^4)$
 (14) $z^4(x^2 + y^2)(x + y)(x - y)$ (15) $(h + k)^2$ (16) $(a + 3)(a - 3)(a + 2)(a - 2)$
 (17) $(h^2 - 6)^2$ (18) $(1 - y)(1 + y + y^2)(1 + y)(1 - y + y^2)$ (19) $4xy$ (20) $x(x^2 + 3x + 3)$

