



Fractions Within Fractions

Nested fractional expressions are expressions that contain fractions within fractions. There are two methods for simplifying nested fractions: performing the division and using an LCM.

METHOD 1: PERFORMING THE DIVISION

The fraction bar is another way of expressing division. If you can turn the numerator and denominator of the big fraction into single little fractions themselves, you can divide those little fractions to simplify the big one.

Example 1: Simplify by performing the division: $\frac{x + \frac{1}{2}}{\frac{2x}{3} - 3}$

Solution: First, we must simplify the numerator and denominator of the big fraction.

$$\begin{aligned}\frac{x + \frac{1}{2}}{\frac{2x}{3} - 3} &= \frac{\frac{2x}{2} + \frac{1}{2}}{\frac{2x}{3} - \frac{9}{3}} \\ &= \frac{\frac{2x+1}{2}}{\frac{2x-9}{3}} \dots\end{aligned}$$

Next, rewrite the big fraction as a division problem, then invert and multiply:

$$\begin{aligned}\dots &= \frac{2x+1}{2} \div \frac{2x-9}{3} \\ &= \frac{2x+1}{2} \times \frac{3}{2x-9} = \frac{6x+3}{4x-18}\end{aligned}$$

METHOD 2: USING AN LCM

A second method involves finding the lowest common denominator for all the little fractions in the problem, and multiplying the top and bottom of the big fraction by that number.

Example 2: Simplify by finding an LCM: $\frac{x + \frac{1}{2}}{\frac{2x}{3} - 3}$

Solution: The denominators of the little fractions in the problem are 2 and 3, which makes the LCM 6.

$$\begin{aligned}\frac{x + \frac{1}{2}}{\frac{2x}{3} - 3} &= \frac{x + \frac{1}{2}}{\frac{2x}{3} - 3} \times \frac{6}{6} \\ \frac{x + \frac{1}{2}}{\frac{2x}{3} - 3} \times \frac{6}{6} &= \frac{6x + \frac{6}{2}}{\frac{12x}{3} - 18}\end{aligned}$$



$$= \frac{6x + 3}{4x - 18}$$

EXERCISES

A. Simplify.

$$1) \frac{x + \frac{1}{3}}{x - \frac{1}{3}}$$

$$6) \frac{\frac{x^2 - 5x + 6}{x^2 - 8x + 12}}{\frac{x^2 - 8x + 15}{2x^2 - 11x - 6}}$$

$$2) \frac{1 + \frac{1}{x}}{1 + \frac{2}{x}}$$

$$7) \frac{\frac{3}{2x+1} - \frac{2}{x+1}}{\frac{5}{x+1} - \frac{2}{x-1}}$$

$$3) \frac{1}{1 + \frac{1}{1 + \frac{1}{x}}}$$

$$8) \frac{x}{x - \frac{x}{x - \frac{x}{x+1}}}$$

$$4) \frac{\frac{1}{x} - \frac{1}{y}}{\frac{2}{x} - \frac{3}{y}}$$

$$9) \frac{\frac{1}{\frac{1}{x} - \frac{1}{y}}}{\frac{1}{x} + \frac{1}{y}}$$

$$5) \frac{\frac{3x}{(x-1)(x+1)}}{\frac{6}{x-1}}$$

$$10) \frac{\frac{y-x}{x-y}}{\frac{\frac{1}{x} - \frac{1}{y}}{\frac{x}{y}}}$$

SOLUTIONS

A. (1) $\frac{3x+1}{3x-1}$ (2) $\frac{x+1}{x+2}$ (3) $\frac{x+1}{2x+1}$ (4) $\frac{y-x}{2y-3x} = \frac{x-y}{3x-2y}$ (5) $\frac{x}{2x+2}$ (6) $\frac{2x+1}{x-5}$ (7) $-\frac{(x-1)^2}{(2x+1)(3x-7)}$

(8) $\frac{x^2}{x^2-x-1}$ (9) $\frac{x^2y^2}{y^2-x^2}$ (10) $2x + 2y$

