Solving Radical Equations

To solve a radical equation with <u>one radical term</u>, follow these steps:

1. Isolate the radical term
2. Square both sides to eliminate the square root sign
3. Solve for x
4. Check for false solutions

Example 1: Solve:

$$\sqrt{x+1} - 3 = 0$$

Solution:

$$\sqrt{x+1} = 3 \quad \text{<Step 1>}$$
$$(\sqrt{x+1})^2 = 3^2 \quad \text{<Step 2>}$$
$$x + 1 = 9$$
$$x = 8 \quad \text{<Step 3>}$$

Check: $\sqrt{8+1} - 3 = \sqrt{9} - 3 = 3 - 3 = 0$ ✓ <Step 4>

To solve a radical equation with <u>two or more radical terms</u>, follow these steps:

1. Isolate one of the radical terms
2. Square both sides
3. If a radical remains, repeat steps 1 and 2 until all radicals are eliminated
4. Solve for x
5. Check possible solutions

Example 2: Solve:

$$\sqrt{x+1} + \sqrt{x+8} = 7$$

Solution:

$$\sqrt{x+1} = 7 - \sqrt{x+8} \quad \text{<Step 1>}$$
$$(\sqrt{x+1})^2 = (7 - \sqrt{x+8})^2 \quad \text{<Step 2>*}$$
$$x + 1 = 49 - 14\sqrt{x+8} + (x + 8)$$
$$x + 1 - 49 - x - 8 = -14\sqrt{x+8} \quad \text{<Step 3: go back to Step 1>}$$
$$-56 = -14\sqrt{x+8}$$
$$4 = \sqrt{x+8}$$
$$4^2 = (\sqrt{x+8})^2 \quad \text{<Step 2>}$$
$$16 = x + 8 \quad \text{<no radical remains, go on!>}$$
$$8 = x \quad \text{<Step 4>}$$

Check: $\sqrt{8+1} + \sqrt{8+8} = \sqrt{9} + \sqrt{16} = 3 + 4 = 7$ ✓ <Step 5>

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***** Remember that you must FOIL an expression like $(7 - \sqrt{x+8})^2$! It's *not* 7 − x + 8.

EXERCISES

A. Solve for x in these problems with only one radical.

1) $\sqrt{x} = x - 6$

2) $\sqrt{x+1} = x - 5$

3) $\sqrt{x^2+15} = 2x + 2$

4) $\sqrt{2x^2-7} - 3x = -5$

5) $\sqrt{2x-3} - x = -9$

6) $\sqrt{3x+4} - x = -2$

B. Solve for x in these problems with more than one radical.

1) $\sqrt{2x+3} = \sqrt{x+5}$

2) $3\sqrt{x-8} = \sqrt{x}$

3) $\sqrt{x+1} + \sqrt{x+6} = 1$

4) $\sqrt{2x-3} - \sqrt{x+2} = 1$

5) $\sqrt{x-9} - \sqrt{x-16} = 1$

6) $\sqrt{5x+5} - \sqrt{x+12} = 1$

7) $\sqrt{4x+13} - \sqrt{6x+7} = 2$

8) $\sqrt{3x-2} - 2 = \sqrt{x-2}$

9) $\sqrt{4x+1} - \sqrt{x-3} = 4$

10) $\sqrt{9x-2} - \sqrt{6x-3} = 1$

SOLUTIONS

A: (1) 9, *not* 4 (2) 8, *not* 3 (3) 1, *not* $-11/3$ (4) 2, $16/7$ (5) 14, *not* 6 (6) 7, *not* 0

B: (1) 2 (2) 9 (3) no solution (*not* 3) (4) 14, *not* 2 (5) 25 (6) 4, *not* $1/4$
(7) −1, *not* 27 (8) 2, 6 (9) 12, *not* $28/9$ (10) $2/3$, 2