VANCOUVER COMMUNITY COLLEGE

Factoring Secrets:

The ac-Method

This worksheet describes a method for factoring expressions of the type $ax^2 + bx + c$. This method is faster than the trial-and-error method.

Example 1: Factor $6x^2 - x - 2$.

Solution:

Step 1: Identify a, b, and c for the expression.

$a = 6, b = -1, c = -2$

Step 2: Multiply a by c (i.e. multiply the coefficient of the x^2 term by the constant term).

$ac = -12$

Step 3: Determine the possible pairs of factors that could yield the product a·c:

$-12 = -1 \times 12 = -2 \times 6 = -3 \times 4 = -4 \times 3 = -6 \times 2 = -12 \times 1$

Step 4: Decide which of the pairs of factors will add up to b.

$b = -1$, and $-4 + 3 = -1$.

Step 5: Replace the middle term by an equivalent expression using the two factors.

$6x^2 - x - 2 = 6x^2 + (-4x + 3x) - 2 = 6x^2 - 4x + 3x - 2$

Step 6: Factor by grouping.

$$\begin{aligned} 6x^2 - 4x + 3x - 2 &= (6x^2 - 4x) + (3x - 2) \\ &= 2x(3x - 2) + 1(3x - 2) \\ &= (2x + 1)(3x - 2) \end{aligned}$$

Example 2: Factor $6x^2 + 19x + 10$.

Solution:

Step 1: Identify a, b, and c for the expression.

$a = 6, b = 19, c = 10$

Step 2: Multiply a by c (i.e. multiply the coefficient of the x^2 term by the constant term).

$ac = 60$

Step 3: Determine the possible pairs of factors that could yield the product a·c:

$60 = 1 \times 60 = 2 \times 30 = 3 \times 10 = 4 \times 15 = 5 \times 12 = 6 \times 10$ and the negative versions of these.

Step 4: Decide which of the pairs of factors will add up to b.

$b = 19$, and $4 + 15 = 19$.

Step 5: Replace the middle term by an equivalent expression using the two factors.

$6x^2 + 19x + 10 = 6x^2 + 15x + 4x + 10$

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Step 6: Factor by grouping.

$$6x^2 + 15x + 4x + 10 = (6x^2 + 15x) + (4x + 10)$$
$$= 3x(2x + 5) + 2(2x + 5)$$
$$= (3x + 2)(2x + 5)$$

Example 3: Factor $6x^2 + x + 1$.

Solution:

Step 1: Identify a, b, and c for the expression.

$a = 6, b = 1, c = 1$

Step 2: Multiply a by c (i.e. multiply the coefficient of the x^2 term by the constant term).

$ac = 6$

Step 3: Determine the possible pairs of factors that could yield the product a·c:

$6 = 1 \times 6 = 2 \times 3 = -1 \times -6 = -2 \times -3$

Step 4: Decide which of the pairs of factors will add up to b.

b = 1, but none of the pairs will add up to 1. This expression is not factorable.

With practice, Steps 3 and 4 could be performed mentally.

EXERCISES

A. Factor, if possible:

1) $6x^2 + 11x + 3$

2) $6x^2 - 7x + 2$

3) $6x^2 + 11x - 10$

4) $10x^2 + 7x - 6$

5) $20x^2 + 7x - 6$

6) $5x^2 - x + 3$

7) $6x^2 + 11x + 4$

8) $6x^2 - 25x + 24$

9) $4x^2 - 4x - 15$

10) $7x^2 - 2x - 3$

SOLUTIONS

A. (1) $(2x + 3)(3x + 1)$ (2) $(3x - 2)(2x - 1)$ (3) $(2x + 5)(3x - 2)$ (4) $(5x + 6)(2x - 1)$
(5) $(4x + 3)(5x - 2)$ (6) Not factorable. (7) $(3x + 4)(2x + 1)$ (8) $(3x - 8)(2x - 3)$
(9) $(2x - 5)(2x + 3)$ (10) Not factorable.