



## Operations with Complex Numbers

Complex numbers are numbers that consist of a real part and an imaginary part. The imaginary part is an  $i$ , (which is  $\sqrt{-1}$ ), with a real number coefficient in front of it. Complex numbers are usually written in the form  $a + bi$ . When you work with complex numbers, you can treat the  $i$  as if it were a variable.

### OPERATIONS

To add or subtract complex numbers, combine the real parts, and combine the coefficients on  $i$ .

*Example 1:* Add  $(1 + 2i) + (3 + 4i)$ .

$$\begin{aligned} \text{Solution: } (1 + 2i) + (3 + 4i) &= (1 + 3) + (2 + 4)i \\ &= 4 + 6i \end{aligned}$$

To multiply complex numbers, foil out the numbers, and then convert  $i^2$  into  $-1$ .

*Example 2:* Multiply  $(1 + 2i) \cdot (3 + 4i)$ .

$$\begin{aligned} \text{Solution: } (1 + 2i) \cdot (3 + 4i) &= 1 \cdot 3 + 1 \cdot 4i + 2i \cdot 3 + 2i \cdot 4i \\ &= 3 + 4i + 6i + 8i^2 \\ &= 3 + 10i + 8 \cdot (-1) \\ &= -5 + 10i \end{aligned}$$

To divide complex numbers, write the division problem as a fraction and then “rationalize” the denominator by multiplying by the conjugate. (The conjugate of  $a + bi$  is  $a - bi$  and vice versa.) Remember to express your final answer in the correct form.

*Example 3:* Divide  $(3 + 4i) \div (1 + 2i)$ .

$$\begin{aligned} \text{Solution: } \frac{3 + 4i}{1 + 2i} &= \frac{3 + 4i}{1 + 2i} \cdot \frac{1 - 2i}{1 - 2i} \\ &= \frac{3 + 4i - 6i - 8i^2}{1 - 2i + 2i - 4i^2} \\ &= \frac{3 - 2i - 8(-1)}{1 - 4(-1)} \\ &= \frac{11 - 2i}{5} \\ &= \frac{11}{5} - \frac{2}{5}i \end{aligned}$$



## EXERCISES

A. Simplify:

1)  $(1 - i) + (2 - 3i)$

8)  $(2 + i) \cdot (2 - 2i)$

2)  $(\frac{1}{2} + \frac{1}{4}i) + (\frac{3}{2} - \frac{5}{4}i)$

9)  $\frac{2+i}{i}$

3)  $(3 - i) - (5 - 2i)$

10)  $\frac{i}{1+i}$

4)  $(\sqrt{2} + 3\sqrt{2}i) - (2\sqrt{2} - \sqrt{2}i)$

11)  $\frac{1+i}{1-i}$

5)  $(1 - i) \cdot (1 + i)$

12)  $\frac{2-i}{2+3i}$

6)  $(1 + \frac{1}{2}i) \cdot (1 - 2i)$

13)  $\frac{\sqrt{2}-i}{\sqrt{2}+i}$

7)  $(3 + i) \cdot (4 - 2i)$

14)  $\frac{3+4i}{3-4i}$

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## SOLUTIONS

A. (1)  $3 - 4i$  (2)  $2 - i$  (3)  $-2 + i$  (4)  $-\sqrt{2} + 4\sqrt{2}i$  (5)  $2$  (6)  $2 - \frac{3}{2}i$  (7)  $14 - 2i$

(8)  $6 - 2i$  (9)  $1 - 2i$  (10)  $\frac{1}{2} + \frac{1}{2}i$  (11)  $i$  (12)  $\frac{1}{13} - \frac{8}{13}i$  (13)  $\frac{1}{3} - \frac{2\sqrt{2}}{3}i$  (14)  $-\frac{7}{25} + \frac{24}{25}i$

