Probability 1:
The Basics

The probability that something happens is a way of describing how likely something is to happen with a number. We cannot say whether it will happen or not, but we can describe how often something has happened in similar situations in the past. When a weather forecast says that there is a 60% chance of rain, this is what the forecaster means: in 60% of the days with conditions similar to today's conditions, it has rained.

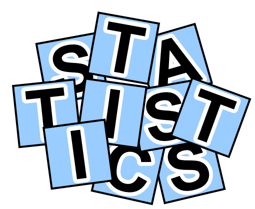
We express probability as percentages between 0% and 100%. An event with a probability of 0% cannot happen; we call it an **impossibility**. An event with a probability of 100% must happen; we call it a **certainty**. We can also describe probabilities with values between 0 and 1, where an impossibility has a probability of 0 and a certainty has a probability of 1. Probabilities cannot be negative and cannot be larger than 100% as a percentage or 1 as a number. (This should make sense — an event cannot have happened more than 100% of the time in the past.)

For events where all outcomes have the same chance of happening, the formula is:

$$P(\text{event}) = \frac{\text{the total number of successes}}{\text{the total number of outcomes}}$$

…where P(event) means "the probability of [event] happening", so P(rolling a 3) means "the probability of rolling a 3"; outcomes are all the things that could happen; and successes are defined as the event you're looking for (whether it's a good thing or not). For quality control in a light bulb factory, where we want to know the probability of a light bulb breaking during manufacturing, a broken bulb is a "success".

Example 1: I select a letter at random from the word STATISTICS. What is the probability that the letter is (a) the letter C? (b) a vowel? (c) a consonant?



Solution: There are 10 letters in the word. The number of outcomes is 10.

(a) Count the number of successes. We want a letter C, so a letter C is a success. There is only one C in the word, so the number of successes is 1.

$\therefore$ P(C) = $\frac{1}{10}$ or 10%

(b) This time we want a vowel. Those are A, E, I, O and U, so any of those letters is a success. There are two I's and an A, so the total number of successes is 3.

$\therefore$ P(vowel) = $\frac{3}{10}$ or 30%

(c) A consonant is any letter that is not a vowel. S, T and C are consonants, so those letters are successes. There are 3 S's, 3 T's and 1 C, for a total of 7 successes.

$\therefore$ P(consonant) = $\frac{7}{10}$ or 70%

The questions in Example 1 demonstrate two important rules of probabilities. Part (b) asked for the probability of choosing a vowel. There are two different vowels in the word, A and I. The number of successes was the number of A's plus the number of I's.

Also, P(vowel) is equal to P(A) + P(I). (Check this for yourself.) This suggests the rule:

> *The probability of any of a group of different events occurring for an experiment is equal to the sum of the probabilities of each event, so long as the events cannot happen at the same time.*

Because we cannot choose an A and an I at the same time — if you choose one, you cannot have chosen the other — the rule applies. We call events like this **mutually exclusive** or **disjoint**. This is an important part of the rule, as Example 2 demonstrates:

Example 2: I select a letter at random from the word "jailbird". What is the probability that the letter (a) is a vowel? (b) has a dot? (c) is a vowel and has a dot (d) either is a vowel or has a dot?

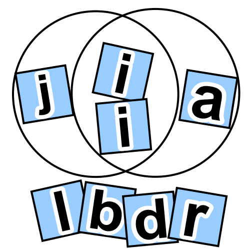
Solution: There are 8 letters in the word. The number of outcomes is 8.

(a) There are 3 vowels in the word, 1 a and 2 i's. The probability is $\frac{3}{8}$.

(b) There are 3 letters with dots in the word, 1 j and 2 i's. The probability is $\frac{3}{8}$.

(c) The only letter that is a vowel and has a dot is i. There are 2 i's. The probability is $\frac{2}{8}$ or $\frac{1}{4}$.

(d) There are 4 letters in the word that are either vowels or which have dots: 1 a, 1 j and 2 i's. The probability is $\frac{4}{8}$ or $\frac{1}{2}$.



The answer to (d) is not $\frac{6}{8}$! If we simply add the answers to (a) and (b), we're counting the outcome of choosing "i" twice. We can, however, add the two and subtract the probability of choosing "i" from the total: $\frac{3}{8} + \frac{3}{8} - \frac{2}{8} = \frac{4}{8} = \frac{1}{2}$. From this, we can extend the previous rule to include outcomes that can happen at the same time:

> *The probability of either of two different events occurring equals the sum of the probabilities of each event minus the probability of them happening together.*

We can use this rule to answer another question for Example 1: What is the probability of choosing either a consonant or a vowel? Since all the letters in STATISTICS are either consonants or vowels, the answer should be $\frac{7}{10} + \frac{3}{10} = \frac{10}{10}$ or 1.This makes sense: all the letters are either consonants or vowels, so we can't help but get a success. This event is a certainty, so the probability should be 1. The next rule, then, is:

> *The probability of an event which accounts for all outcomes in a situation is 1. In other words, the sum of the probabilities of all events for a situation is 1.*

This rule suggests a useful shortcut for answering questions with the word "not" in them. The probability of choosing a consonant is the same as the probability of not choosing a vowel. Since we've already calculated the probability of choosing a vowel to be $\frac{3}{10}$, and all probabilities must add up to 1, the probability of not choosing a vowel must be $\frac{7}{10}$.

> *The probability of an event expressed as some other event not happening is 1 minus the probability of the other event.*

In symbols, the rules are:

P(A or B) = P(A) + P(B), if and only if A and B are disjoint. [The Addition Rule]
P(A or B) = P(A) + P(B) − P(A and B) [The General Addition Rule]
P(A) + P(B) + P(C) + … = 1
P(not A) = 1 − P(A)

EXERCISES

A. 1) There are three colours of socks in a drawer: blue, black and white. If you have a 55% chance of drawing a white sock at random and a 25% chance of drawing a black sock at random, what are your chances of drawing a blue sock?

2)If a weatherman says that there is a 70% chance of heavy rain tomorrow and a 55% chance of high wind, is the forecast wrong? What does such a forecast mean?

3) Your coach tells your team, “If you give 110% on the field, you have a 110% chance of winning!” Explain what such a statement would mean in terms of probability and why it is wrong.

B. A deck of cards has 13 ranks (A, 2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K) and four suits (♥, ♦, ♣, ♠), with each configuration of rank and suit occurring only once. The deck is thoroughly shuffled, randomizing the cards. What is the probability of drawing:

1) an 8 from the deck?

2) a ♠ (spade) from the deck?

3) a J, Q, or K (a face card) from the deck?

4) a 10 or a ♥ (heart) from the deck?

5) drawing a 7 if you’ve already taken 8♠, 7♠, J♦ and 7♥ from the deck?

C. A pair of dice, one red and one green, are rolled.

1) Complete the table. How many outcomes are there?

	1	2	3	4	5	6
1	**2**	**3**				
2	**3**	**4**				
3						
4						
5						
6						

What is the probability of rolling:

2) a total of 8 exactly?

3) a total of 8 or more?

4) a ⚄ on the red die? The green die?

5) a ⚄ on either die?

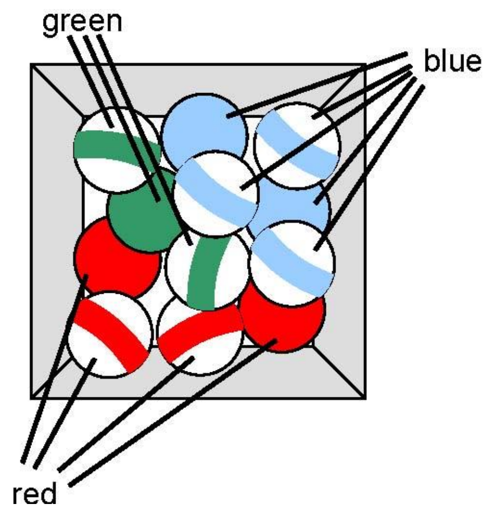
D. The board game “Formula D” comes with a six-sided die where one side has a 2, two sides have a 3 and three sides have a 4. A statistics student considers the problem of rolling two dice like this and tries to calculate the probability of rolling a total of 6. He makes a table like the one at the right and gets an answer of $\frac{3}{9}$ or $\frac{1}{3}$.
This is not the right answer.

	2	3	4
2	**4**	**5**	**6**
3	**5**	**6**	**7**
4	**6**	**7**	**8**

1) What is wrong with his method?

2) What is the correct answer?

E. A box contains the set of balls shown to the right. The box is shaken, and one ball is chosen at random. What is the probability of choosing:



1) a ball with red on it?
2) a ball with blue on it?
3) a ball with either red or blue on it?
4) a striped ball?
5) a solid ball?
6) a striped or solid ball?
7) a ball with red on it or a striped ball?
8) a ball that is not green striped?

SOLUTIONS

A: (1) 20% (2) The forecast is not necessarily wrong wince it is possible to have both rain and wind on the same day. The forecast means that on days with conditions similar to the current weather conditions, 70% of the time it rained the next day and 55% of the time there was high wind the next day. (3) It would mean, in the coach’s experience, when a team has given 110% on the field, the team has won 110% of the time. This is not possible since there’s no such thing as “110% of the time”.

B: (1) $\frac{4}{52} = \frac{1}{13}$ (2) $\frac{13}{52} = \frac{1}{4}$ (3) $\frac{12}{52} = \frac{3}{13}$ (4) $\frac{16}{52} = \frac{4}{13}$ (5) $\frac{2}{48} = \frac{1}{24}$

C: (1) 36 outcomes (2) $\frac{5}{36}$ (3) $\frac{15}{36} = \frac{5}{9}$ (4) $\frac{1}{6}$ in both cases (5) $\frac{6}{36} + \frac{6}{36} - \frac{1}{36} = \frac{11}{36}$

D: (1) The student’s method does not account for the fact that each number of the dice has a different chance of occurring. This means that each of the totals in the table has a different chance of happening, so their individual probabilities are not all $\frac{1}{9}$.
(2) If a 6×6 table like the one for question C were constructed, we get $\frac{10}{36} = \frac{5}{18}$.

E: (1) $\frac{4}{12} = \frac{1}{3}$ (2) $\frac{5}{12}$ (3) $\frac{4}{12} + \frac{5}{12} = \frac{9}{12} = \frac{3}{4}$ (4) $\frac{7}{12}$ (5) $\frac{5}{12}$ (6) 1
(7) $\frac{4}{12} + \frac{7}{12} - \frac{2}{12} = \frac{9}{12} = \frac{3}{4}$ (8) $1 - \frac{2}{12} = \frac{10}{12} = \frac{5}{6}$