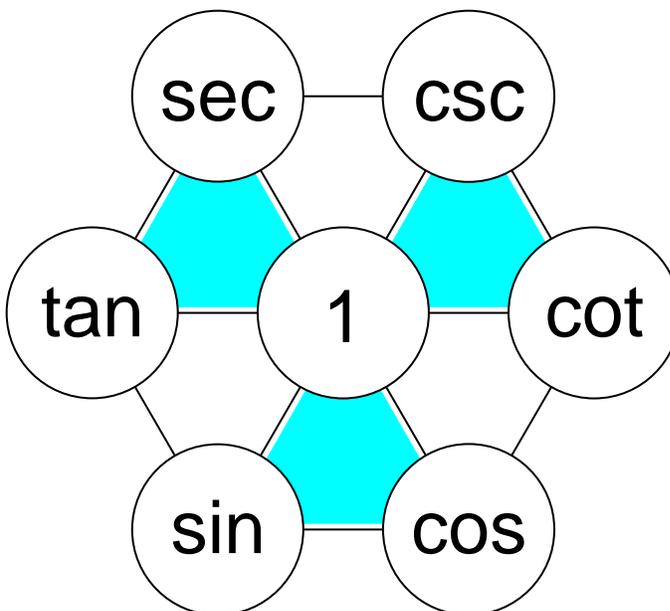


32 Identities in 32 Seconds

This diagram is easy to remember, easy to draw, and will help you recall 32 identities in a short time. To construct the diagram, just do this:

- [1] Draw a hexagon.
- [2] Place the **sin function** in the lower left corner.
- [3] Place the other trig functions **counterclockwise** from sine in **alphabetical order** (i.e. cos, cot, csc, sec, tan).
- [4] Put a 1 in the middle.



The identities can be read from the diagram as follows:

- [1] Functions at opposite ends of the diagonals are reciprocals. This means that if you multiply the two functions at the ends of the diagonals, you get the "1" that is in the middle.

$$(1) (\sec x)(\cos x) = 1 \quad (2) (\sin x)(\csc x) = 1 \quad (3) (\tan x)(\cot x) = 1$$

- [2] Any function is equal to the product of the two functions adjacent to it.

$$\begin{array}{ll} (4) \sin x = (\cos x)(\tan x) & (7) \csc x = (\cot x)(\sec x) \\ (5) \cos x = (\sin x)(\cot x) & (8) \sec x = (\csc x)(\tan x) \\ (6) \cot x = (\cos x)(\csc x) & (9) \tan x = (\sec x)(\sin x) \end{array}$$

- [3] For any three consecutive functions around the hexagon in either direction, the first function equals the second function divided by the third function.

$$\begin{array}{ll} (10) \sin x = \frac{\cos x}{\cot x} & (12) \cos x = \frac{\cot x}{\csc x} \\ (11) \sin x = \frac{\tan x}{\sec x} & (13) \cos x = \frac{\sin x}{\tan x} \end{array}$$



$$(14) \cot x = \frac{\csc x}{\sec x}$$

$$(15) \cot x = \frac{\cos x}{\sin x}$$

$$(16) \csc x = \frac{\sec x}{\tan x}$$

$$(17) \csc x = \frac{\cot x}{\cos x}$$

$$(18) \sec x = \frac{\csc x}{\cot x}$$

$$(19) \sec x = \frac{\tan x}{\sin x}$$

$$(20) \tan x = \frac{\sin x}{\cos x}$$

$$(21) \tan x = \frac{\sec x}{\csc x}$$

[4] The product of any three alternating functions is 1.

$$(22) (\cos x)(\csc x)(\tan x) = 1$$

$$(23) (\sin x)(\cot x)(\sec x) = 1$$

[5] Note the three shaded triangles within the hexagon. They can be used to recall the Pythagorean identities. The sum of the squares of the functions at the *base* of the triangle is equal to the square of the function at the *top*.

$$(24) \sec^2 x = \tan^2 x + 1$$

$$(26) 1 = \sin^2 x + \cos^2 x$$

$$(25) \csc^2 x = 1 + \cot^2 x$$

[6] The cofunction identities can be reconstructed from the rows within the hexagon.

$$(27) \sec(\pi/2 - x) = \csc x$$

$$(30) \csc(\pi/2 - x) = \sec x$$

$$(28) \tan(\pi/2 - x) = \cot x$$

$$(31) \cot(\pi/2 - x) = \tan x$$

$$(29) \sin(\pi/2 - x) = \cos x$$

$$(32) \cos(\pi/2 - x) = \sin x$$

OPTIONAL: CALCULUS APPLICATIONS

If you get into calculus, the hexagon can also help you to remember the derivatives of the six trig functions. Draw a vertical line through the middle of the hexagon. The functions on the left of the hexagon have positive derivatives and the functions on the right have negative derivatives.

The derivatives on the bottom row are easy; they point to each other:

$$(a) \frac{d}{dx} (\sin x) = \cos x$$

$$(b) \frac{d}{dx} (\cos x) = -\sin x$$

For the derivatives of the middle row functions, take the function above and square it:

$$(c) \frac{d}{dx} (\tan x) = \sec^2 x$$

$$(d) \frac{d}{dx} (\cot x) = -\csc^2 x$$

For the derivatives of the top row functions, take the original function and multiply by the function below:

$$(e) \frac{d}{dx} (\sec x) = \sec x \tan x$$

$$(f) \frac{d}{dx} (\csc x) = -\csc x \cot x$$

