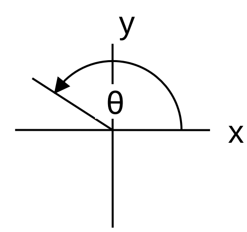
Directions of Travel

Many problems in trigonometry and physics involve objects that are moving in straight lines, but not at neat 90° angles to each other. There are many ways to express these directions. Some of them are standards in math and science, and others are used in real-life situations in various occupations. This worksheet will help you to distinguish between them and use them in problems.

DIRECTED ANGLES

Directed angles are used in trigonometry. It's a standard that derives from the unit circle and makes use of the coordinate system of the plane. These angles are expressed in radians.

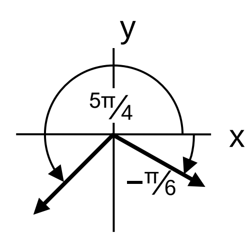


To find a directed angle, start from the **positive x-axis** and move **counterclockwise**. The positive x-axis marks 0 rad, and the negative x-axis marks π rad. 2π takes us back to the positive x-axis.

Example 1: Draw an angle of a) $\frac{5\pi}{4}$ rad; b) $-\frac{\pi}{6}$ rad; c) $\frac{20\pi}{3}$ rad.

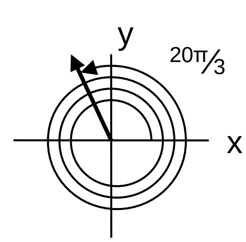
Solution: When you start learning how to work with radians, it may be useful to convert these angles to degrees, but it's better to practice thinking in radians. We can start by figuring out how our angle compares to π and 2π so that we have some idea what the angle looks like.

a) $\frac{5\pi}{4}$ is higher than π but less than 2π (because $\frac{5}{4}$ is higher than 1 but less than 2), so the angle lies below the x-axis. Since $\frac{5}{4}$ is equivalent to $1\frac{1}{4}$, the angle is π plus $\frac{1}{4}$ of the way to 2π.



b) $-\frac{\pi}{6}$ is less than 0, so instead of moving counterclockwise, we move clockwise the same amount we'd be moving for $+\frac{\pi}{6}$. This is one of the angles we should know. $-\frac{\pi}{6}$ is equivalent to 30° below the positive x-axis.

c) $\frac{20\pi}{3}$ is larger than 2π (= $\frac{6\pi}{3}$), so this angle is equivalent to some other angle between 0 and 2π, which is the range we usually use with angles in radians. We can find out what that angle is by subtracting 2π from the angle until our answer is within this range (or adding 2π to the angle if the angle we're given is negative). The answer we get is called the **reference angle**.



$$\frac{20\pi}{3} - 3 \times \frac{6\pi}{3} = \frac{20\pi}{3} - \frac{18\pi}{3} = \frac{2\pi}{3}$$

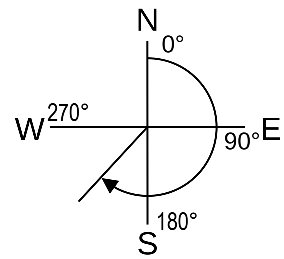
The reference angle is between 0 and π, so it's above the x-axis, $\frac{2}{3}$ of the way to π.

Authored by Darren Rigby

(This is another angle whose measure in degrees we should know; it's 120°.)

HEADINGS, BEARINGS AND COURSES

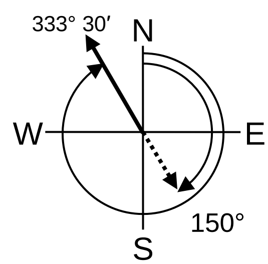
Planes and boats travel on headings or bearings. It's a numerical way of expressing vague directions like "southwest" when something more specific is needed.



To interpret a bearing, start at **north** and go **clockwise**. Bearings can be anywhere from 0° to 359° 59′ 59″. The symbol ′ means "minutes", and the symbol ″ means "seconds". Just as with an hour, every degree can be divided into 60 minutes, and every minute into 60 seconds.

Example 2: Draw headings of a) 150° and b) 333° 30′. Which general direction of travel do they represent (northwest, northeast, southwest, southeast)?

Solution: a) 150° is between 90° (east) and 180° (south), so the direction is southeast. 150° − 90° = 60°, so it's 60° past 90°. 150° is the dashed arrow in the diagram.



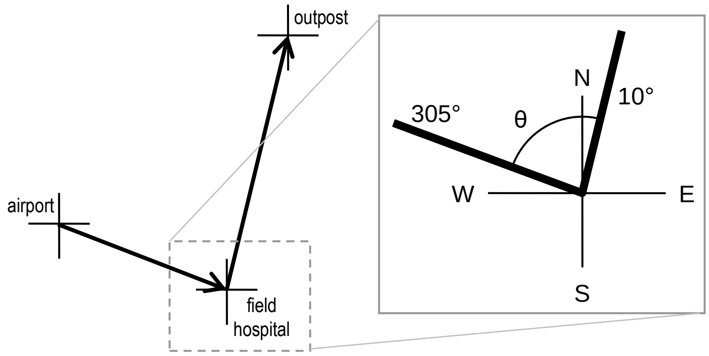
b) 333° 30′ is between 270° (west) and 360° (north), so the direction is northwest. 333° 30′ − 270° = 63° 30′, so it's a little over 60° past due west. This bearing is the solid arrow in the diagram.

Example 3: A plane carrying supplies in the Arctic leaves Yellowknife airport on a bearing of 125°, and travels in a straight line to a field hospital. After it drops off supplies at the hospital, it takes off on a bearing of 10° to pick up mail from an outpost. What is the angle between the plane's course on arrival and course of departure at the hospital?

Solution: This type of problem usually appears in trigonometry, where the real idea is to figure out what bearing the plane needs to go on to get back to its airport, but the first critical step is finding the angle at the hospital.

A diagram is very useful. The diagram at the top of the next page shows the three locations.

The bearing away from the airport is 125°. This is *not* the bearing at the field hospital, since as you can see in the diagram, 125° is a southeastern direction, but the plane arrives at the field hospital from a northwestern direction! We correct this by thinking of the plane's incoming trajectory seen from the field hospital as being 180° away (the opposite way) from its trajectory away from the airport. **We either add or subtract 180° from any bearing we are given if we are interested in its arrival** rather than its departure. We can't subtract (since a bearing cannot be negative), so we add: 125° + 180° = 305°. Now we need to find θ, the angle between 305° and 10°.
This angle includes the due-north line, so the bearings "turn the corner", as it were. In any other problem, we'd just subtract these bearings, and that would be the answer, but here, we'd get the reflex angle going the other way. That's not helpful. To correct for this, we add 360° to the 10° to get 370°, and then we subtract these: θ = 370° − 305° =

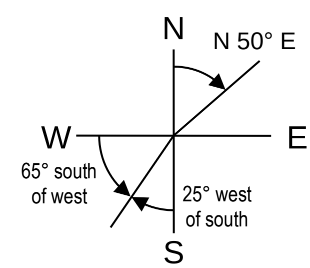


65°.

Another way to get the angle in a problem like this is to compare each line of travel in the angle to the four compass points, and build the desired angle in pieces. The part of θ that is between the 305°-line and due North is 360° − 305° = 55°. The part of θ that is between due North and the 10°-line is 10°. When we put these parts together, we get 55° + 10° = 65°, which is the same answer we got with the other method.

COMPASS-POINT NOTATION

Compass-point notation is used by hikers, and is common in physics problems. It tells you the angle of a direction compared to the four main compass directions.



You will sometimes see a notation like "N 50° E". This form starts with **one direction** and goes **toward the second direction**. (If you're doing a Math 12 problem, north or south must always be first. In physics problems, east or west can be first.) This direction means start at due North and adjust 50° to the east.

You'll also encounter a second notation like "65° south of west". In this case, you start from the **second direction mentioned** and move toward the **first direction mentioned**. If this is still confusing, you can think of a direction like "65° south of west" as meaning "65° _to the_ south of _due_ West". Using the complementary angle and reversing the order of the compass points will give you a second direction that means the same thing. (The angle measure between due South and due West is 90°, so coming 65° from the west is equivalent to coming 90° − 65° = 25° from the south.)

Example 4: Convert: a) 20° east of south to a bearing, and b) 282° to a compass point.

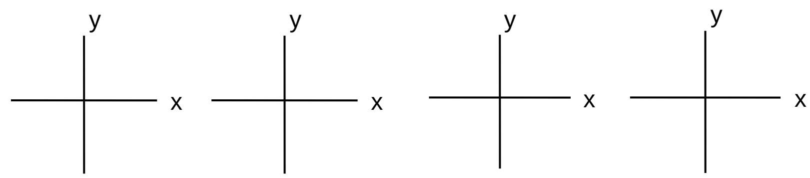
Solution: a) "20° east of south" means 20° to the east of due South. When we do a bearing, we start at North and go clockwise. Starting from South and going to the east is counterclockwise, which is the wrong way for a bearing. South is a bearing of 180°, and since we need to go counterclockwise, we subtract 20° to get 160°. (Going clockwise, bearings get larger, so going counterclockwise, they get smaller, so we subtract.)

b) We examine the bearings at each of the compass points: North is 0°, East is 90°, South is 180°, and West is 270°. We have to go west, and then 282° − 270° = 12° further toward the north. A bearing of 282° is thus 12° to the north of due West, or 12° north of west.

EXERCISES

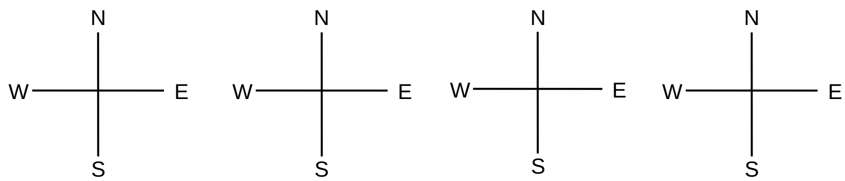
A. Draw these angles. (You don't need to include the arc, just the arrow leading away from origin.) Identify the reference angles:

1) $\frac{3\pi}{2}$
2) $-\frac{19\pi}{6}$
3) $\frac{17\pi}{4}$
4) $\frac{1000\pi}{2}$



B. Draw these headings. State what general direction each represents:

1) 355°
2) 200°
3) 115°
4) 270°



C. Convert:

1) bearing 35° to a compass point
2) S 20° E to a bearing
3) 55° west of north to a bearing
4) bearing 225° to a compass point
5) bearing 104° 25′ to a compass point
6) 35° 15′ south of west to a bearing

D. Determine the angle between these courses:

1) a tanker leaves port for an oil rig at a bearing of 130° and departs the rig at 225°

2) a jet airliner leaves YYZ on runway 305 to Vancouver and takes off from runway 195 at Vancouver bound for Seattle [*Hint: Runways are numbered after the bearing they are on from the airport.*]

3) a water bomber leaves an air base on a bearing of 117° 40′ to get water from a lake, and goes directly to a forest fire on a bearing of 28° 15′

SOLUTIONS

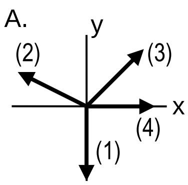
A. See diagram: 1) $\frac{3\pi}{2}$ 2) $\frac{5\pi}{6}$ 3) $\frac{\pi}{4}$ 4) 0

B. See diagram: 1) northwest 2) southwest 3) southeast 4) west

C. Other answers possible: 1) 35° east of north 2) 160° 3) 305° 4) 45° west of south 5) 14° 25′ south of east 6) 234° 45′

D. 1) 85° 2) 70° 3) 90° 35′

A.



B.