



Logic Puzzles

A fun way to test your ability to apply logical thought to a problem is to solve logic puzzles. Your assignments and textbook contain examples of **deductive logic puzzles**: puzzles where you have a series of clues (which you may accept as premises) that can lead to a unique mapping of one set of items to another. We can't use truth tables to answer questions like this (...well, we *could* if we made every combination of answers a single statement, and added conclusions preventing duplication, but this gets to be a large problem quickly!). The simplest way to solve it is by using a grid and logical laws.

Example 1: Cristobal, Eduardo, Gabriela and Lucia are in VCC's International Culinary program. They come from Argentina, Bolivia, Panama and Spain, with one student coming from each country.

1. The statement "If Cristobal is from Panama, then Gabriela is from Argentina," is false.
2. The four students are Lucia, Cristobal, the Spanish student, and the Bolivian student.
3. Eduardo is better at baking than the student from Bolivia.

Where is each student from?

Solution: We can start this problem by making a grid:

	A	B	P	S
C				
E				
G				
L				

The puzzle constructor has been kind — all our elements start with different letters of the alphabet, so we can easily abbreviate everything. Our students, Cristobal, Eduardo, Gabriela and Lucia are down one side, and the countries are across the top: Argentina, Bolivia, Panama and Spain.

We can fill the information we know into the grid. We can use a check mark for correct matches and an X for incorrect matches. Since we also want to be able to show our work, we should also record the logical process we use to get our answers.

Notice that in this question we have four students and four countries. We are told in the instructions that each person comes from a different country. That means that, for example, if we know Cristobal is from Argentina, of course he's not from the other countries, but also, no one else is from Argentina.

Clue 3 is a little strange. What does Eduardo's baking ability have to do with where he's from? If Eduardo is *better* than the student from Bolivia, he can't *be* the student from Bolivia. Statements like this can be common in logic puzzles, hiding information behind other information. We can enter that information into the grid, and we'll write down how we know it.



	A	B	P	S
C				
E		X		
G				
L				

Eduardo is not from Bolivia (clue 3).

Just as in a direct proof, we should say where we are getting our information from.

Clue 2 is structured the same way. If the list includes all four people, then no one appears on the list twice. That means Lucia isn't from Spain or Bolivia, and neither is Cristobal. We can cross off four more squares in our grid.

Neither Lucia nor Cristobal is from Spain or Bolivia (clue 2).

Look at the column for Bolivia. So far, we know that Cristobal, Eduardo and Lucia aren't from Bolivia, so Gabriela must be. When we block out all but one of the squares in a row or column, we can mark the remaining square with a check mark. Also, we can block out the other squares in Gabriela's row — she's not from any other country.

The student from Bolivia is not Eduardo, Lucia or Cristobal, so it must be Gabriela (Instructions: there is one student from each country). Gabriela is not from Argentina, Panama or Spain (Instructions: Each student is from a different country).

	A	B	P	S
C		X		X
E		X		
G				
L		X		X

	A	B	P	S
C		X		X
E		X		
G	X	✓	X	X
L		X		X

This creates a new column where we may draw a conclusion: we know Eduardo is from Spain.

The student from Spain is not Lucia, Cristobal or Gabriela, so it must be Eduardo (Instructions: there is one student from each country). Eduardo is not from Argentina or Panama (Instructions: Each student is from a different country).

This leaves us with two people and two countries: Cristobal and Lucia from Argentina and Panama. We also haven't used Clue 1:

The statement "If Cristobal is from Panama, then Gabriela is from Argentina," is false. If the statement itself is false, then its negation is true. If we can negate the statement, we can mine it for information.

Let c be the proposition "Cristobal is from Panama" and g be the proposition "Gabriela is from Argentina". Then Clue 1 says:

- ① $\neg(c \rightarrow g)$
- ② $\Leftrightarrow \neg(\neg c \vee g)$ Implication, 1
- ③ $\Leftrightarrow \neg\neg c \wedge \neg g$ De Morgan's, 2
- ④ $\Leftrightarrow c \wedge \neg g$ Dbl Negation, 3
- ⑤ $\therefore c$ Conjunctive Simplification, 4

So we know c , Cristobal is from Panama. This gives us the solution to the problem.
Cristobal is from Panama, Eduardo is from Spain, Gabriela is from Bolivia, and Lucia is from Argentina.

Remember, the best way to use the grids is to record information you're sure of, not necessarily to indicate what could be true.

	A	B	P	S
C		X		X
E	X	X	X	✓
G	X	✓	X	X
L		X		X



EXERCISES

A. Ian, Veronica, Mohammad and Cariad have been hired on at a software company. Their new job titles are programmer, QA specialist, UI designer and technical writer. Each person is only doing one job.

- 1) It is not the case that Ian is the technical writer and Veronica is the QA specialist.
- 2) The statement, "If Ian is not the QA specialist, then Cariad is the programmer," is false.
- 3) Mohammad is either the programmer or the technical writer.
- 4) The UI designer is either Mohammad or Cariad.

Who was hired to each position? Write a full solution.

	P	Q	T	U
C				
I				
M				
V				

B. Three customers at the electronics store (Amanda, Stephan and Nabeela) each bought a smart phone (one, an iPhone, one an Android phone and one a Blackberry) and each bought a different accessory (a case, an extra charger and an extra battery).

- 1) No two customers bought the same type of accessories or phones.
- 2) If Nabeela bought the iPhone, then Stephan bought the battery.
- 3) Stephan did not buy the charger.
- 4) The person who bought the battery bought the Android phone.
- 5) If the person who bought the Blackberry didn't buy the battery, then Amanda bought it.

What did each customer buy? Write a full solution.

	iP	A	Bb	b	ca	ch
A						
N						
S						
b						
ca						
ch						



SOLUTIONS

A: Mohammad is either the programmer or the technical writer (clue 3). Therefore Mohammad is not the QA specialist, and he's not the UI designer. (Instr.: Each person...) The UI designer is either Mohammad or Cariad (clue 4). Since we know it's not Mohammad, Cariad must be the UI designer.

Clue 2 says that an implication statement is wrong. Implication statements are wrong only if the supposition is true and the conclusion is false. Therefore Ian is not the QA specialist, and Cariad is not the programmer. We know Cariad is not the QA specialist (paragraph 1), Ian is not the QA specialist (clue 2), and Mohammad is not the QA specialist (paragraph 1). Since someone is the QA specialist (Instr.: Their new...), it must be Veronica.

Clue 1 says it is not the case the Ian is the technical writer and Veronica is the QA specialist. Veronica is the QA Specialist (paragraph 2). If Ian were the technical writer, Clue 1 would be false, so Ian is not the technical writer. Ian is also not the QA specialist (clue 2) or the UI designer (since Cariad is, paragraph 1). He must be the programmer (Instr.: Their new...)

By process of elimination, Mohammad is the technical writer.

Therefore, Cariad is the UI designer, Ian is the programmer, Mohammad is the technical writer and Veronica is the QA specialist.

B: The person who bought the battery bought the Android phone (clue 5). This means that the person who bought the BlackBerry didn't buy the battery (clue 1).

Clue 6 says that if the person who bought the BlackBerry didn't buy the battery, then Amanda did. By Modus Ponens, we know Amanda did buy the battery. This means she did not buy the charger, and Stephan didn't buy the battery (clue 1).

Stephan did not buy the charger (clue 4). If Stephan didn't buy it and Amanda didn't buy it, then Nabeela bought the charger (Instr.) Stephan didn't buy the battery or the charger, so he bought the case.

The person who bought the battery bought the Android phone (clue 5). Amanda bought the battery (paragraph 2). Thus Amanda bought the Android phone. This means she did not buy the iPhone (Instr.), and Nabeela did not buy the Android phone (clue 1).

Stephan did not buy the battery (paragraph 2). Clue 2 says that if Nabeela bought the iPhone, then Stephan bought the battery. My Modus Tollens, Nabeela did not buy the iPhone. Nabeela didn't buy the Android phone either (paragraph 3), so she bought the BlackBerry. Neither Nabeela nor Amanda bought the iPhone (paragraph 4), so Stephan did (Instr.).

Therefore, Amanda bought the Android phone and an extra battery, Nabeela bought the BlackBerry and an extra charger, and Stephan bought the iPhone and a case.

