Break Free of Your Calculator!

Today, using a calculator is a critical part of doing work in math and science. There's a greater possibility for making mistakes when working out complicated problems by hand, and using a calculator is faster. On the other hand, some calculations are so simple that a calculator shouldn't be necessary. If you have to add 3 + 4, you shouldn't need to reach for your calculator. As we get into more complex problems in math and science, we rely on the calculator more and more, and soon it just becomes easier to let the calculator do the work for us. We start to get lazy about arithmetic. This worksheet will describe some of the dangers of relying on your calculator for every calculation, and it will show you some tricks to help boost your mental math skills.

WHY NOT LET THE CALCULATOR DO IT ALL?

Calculators are fast and accurate, but there are two flaws with using the calculator for everything: it does *exactly* what you tell it to do, and it allows you to stop thinking.

While calculators never make mistakes, people still do, and *you* have to tell the calculator what to do. No matter how skilled you become at math, you will still do crazy things like write 15 + 15 = 20 and then go on with the rest of your problem. If you have an error, you'll still get an answer, but it won't be right. Math is like this.

If you type something and yuo have manny tpying mistakes, you wil noticwe then assoon as you raed them, becuase they stikc out. You probably understand the last sentence, but it was painful to read because of all the errors. It is useful to develop the same sense of "wrongness" when we look over a math problem. This ties in to the other danger of letting the calculator do it all: it lets us stop thinking.

"Not thinking" sounds good, doesn't it? Chemistry, physics and calculus are hard enough without having to worry about doing arithmetic too! I passed Grade 3 a long time ago! Why should I need to do math in my head? Why should I have to *think* about everything? The reason is that when you're in math class, you're not just learning math. You're learning how to solve problems. Solving problems requires thinking. If we get lazy and stop thinking, we make more mistakes. Isn't it really irritating when you get a test back and discover that you knew the concepts and equations… but your calculations were wrong because you forgot a negative sign? Being more careful about your math improves your marks, but you can only catch obvious mistakes if you're thinking. So what can you do to help yourself get better at the simple math?

THE SANITY CHECK

Even on a question where you can't tell what the answer is just by looking, you can often get an idea of how big the answer is, or whether you expect the answer to be positive or negative. Comparing the answer you get from a problem to your expectations of the answer is sometimes called a **sanity check**.

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For example, I once had a student, a trades worker, who had been asked to calculate the volume of a box: 2 m × 150 cm × 200 cm. He made a mistake: he did not convert all the measurements to the same unit. Instead, he just multiplied: 2 × 150 × 200 = 60 000. I asked him, “60 000 what? What units would that be?” He thought a second, then said “Metres cubed?” I told him to do a sanity check. He knew from his work experience how big such a box should be — only a little larger than a desk — and looked at his answer. He started laughing. Sixty thousand cubic metres is the size of a warehouse. He still didn’t know what the answer was, but he knew the answer he had was wrong!

Doing a sanity check on an answer can help you find a silly mistake, like typing the wrong number into a calculator or multiplying where you should be dividing. If the answer you get doesn’t match what the question asks for (negative amounts of time, a number that’s small when it should be large, . . .), a silly mistake may be responsible.

TIPS FOR MENTAL MATH

There are some strategies you can use to help you do mental math faster and more accurately. Most of these strategies allow you to do calculations that would be hard to do in your head and make them easier. This section assumes you can do very simple calculations, like adding one-digit numbers and multiplying one-digit numbers.

Addition and Subtraction — Start from the Left

In North American schools we are taught to add starting with the column on the right:

$$\begin{array}{r} 23 \\ +47 \\ \hline \end{array}$$ We add the 3 and 7 to get 10, and carry the 1…

You will probably find it easier to add each number, piece by piece. To do the problem above, think: *The first number is 23. I add 47, which is 40 and 7. Twenty-three plus forty is sixty-three, plus seven more is seventy.* Repeating the number to yourself in your head will help you to remember what total you’re at for each step of the calculation.

You can add larger numbers this way, such as 285 + 613: *Start with 285. I add 613, six hundred, and ten, and three, Two hundred eighty-five plus six hundred is eight hundred eighty-five, plus ten is eight hundred ninety-five, plus three is eight hundred ninety-eight.*

You can even add entire columns of numbers like this. To add 285 + 153 + 932 + 361: Start with 285, plus 100 is 385, plus 900 is 1285, plus 300 is 1585. Plus 50 is 1635, plus 30 is 1665, plus 60 is 1725. Plus 3 is 1728, plus 2 is 1745, plus 1 is 1746.

Subtracting works the same, and is even easier when you start from the left. To subtract 538 − 382: *Start with 538. Minus three hundred is two thirty-eight, minus eighty is (23 − 8 = 15) one fifty-eight, minus two is 156.* This technique is hard at first, but if you’re breaking your addiction to the calculator, it’s going to take some exercise to get those mental muscles limber again. Don’t give up!

Adding and Subtracting — Nine Is Almost Ten

To add 9 (or 90, or 900…) you can add 10 (or 100, or 1000,…), then subtract 1 at the end. 25 + 9 is the same as 25 + 10 − 1, since 10 − 1 = 9. So 25 + 10 is 35, minus 1 is 34.

To subtract 9, subtract 10 and then add 1 at the end. 56 − 9 is the same as 56 minus 10, 46, plus 1, 47.

Multiplying by 9 — Nine Is Still Almost Ten

I'm sure you already know how to multiply a number by ten: you add a zero to the end. For two-digit numbers, maybe three if you practice, you can more easily multiply by 9 by multiplying by 10 and then subtracting the original factor. To multiply 67 × 9: 67 × 10 = 670. Now use your new mental skills to subtract the original factor. 670 − 67: *Six hundred seventy minus sixty is six hundred ten, and six hundred ten minus seven is 603.* Why does this work? If we replace the 9 by 10 − 1, and distribute, we get:

$$\begin{aligned} 67 \times 9 &= 67 \times (10 - 1) \\ &= 67 \times 10 - 67 \times 1 \\ &= 67 \times 10 - 67 \end{aligned}$$ …and that's math you can do in your head.

Multiplying Two-Digit Numbers — Algebra Still Works Without Variables

You can "distribute" two-digit numbers. Again, start with the big numbers and work your way down. To do 48 × 7, break the two-digit number down into the meanings of its digits:

$$48 \times 7 = (40 + 8) \times 7$$

40 × 7 is 4 × 7 plus a 0… 280. 8 × 7 is 56. 280 plus 50 is 330, plus 6 is 336.

Dividing — Sometimes It's Easier Than It Looks

Not every problem lends itself well to mental math, but it's worth checking for those problems that do. Some division problems can be broken down mentally. If I have to divide 1228 by 4, I look at the bigger number first. I see 12 next to 28, and both of those are divisible by 4. 1200 divided by 4 is 300, and 28 divided by 4 is 7, so 1228 ÷ 4 = 307. If you can break up a division problem this way, you can divide in your head.

Dividing by 2 — You Know Why They Call It "Long Division"?

Because there's short division! In school, the emphasis shifted to long division, but short division is a technique that can be done mentally, or with pencil and paper.

To divide by 2, you just need to keep track of odd and even numbers within your problem. To do 6574 ÷ 2: 6 ÷ 2 = 3, so write 3. 5 ÷ 2 = 2, with 1 remainder. Write 2 (we now have 32…) and mentally put the 1 remainder on the next digit: the 7 becomes 17. 17 ÷ 2 = 8 R 1 (328…). Put the 1 with the next digit: 14 ÷ 2 = 7. 6574 ÷ 2 = 3287.

Division by any other one-digit number can be done this way too.

Multiplying by 5 — Five Is Half of Ten

Now that you can divide by 2, you can multiply by 5. To do 638 × 5, multiply by 10 and divide the result by 2, since 638 × 5 = 638 × 10 ÷ 2. 6380 ÷ 2 goes like this: 6 ÷ 2 = 3, so write 3. 3 ÷ 2 is 1 R 1. (31…) 18 ÷ 2 = 9 (319…) and 0 ÷ 2 = 0, so 638 × 5 = 3190.

EXERCISES

A. Calculate these without a calculator. Check each answer as you do it. The two letters after each problem will tell you where to find the solution. [**Ac** is in the first row (**A**) and the third column (**c**).]

1) 42 + 31 (**Dg**)
2) 67 + 52 (**Ec**)
3) 14 + 19 (**Aj**)
4) 235 + 124 (**Cf**)
5) 503 + 382 (**Fh**)
6) 472 + 317 (**Bc**)
7) 624 + 823 (**Dk**)
8) 905 + 465 (**Ag**)
9) 649 + 472 (**Dd**)
10) 736 + 857 (**Fe**)
11) 83 − 42 (**Be**)
12) 97 − 52 (**Da**)
13) 76 − 31 (**Ci**)
14) 85 − 38 (**Fj**)
15) 125 − 77 (**Bh**)
16) 123 + 624 + 325 (**Cℓ**)
17) 237 + 486 + 919 (**Ef**)
18) 2475 + 5371 (**Bb**)
19) 35 + 9 (**Fb**)
20) 27 + 9 (**Eh**)
21) 167 + 90 (**Ad**)
22) 382 + 99 (**Ei**)
23) 42 − 9 (**Fℓ**)
24) 83 − 9 (**Dj**)
25) 114 − 90 (**Cb**)
26) 326 − 99 (**Fa**)
27) 23 × 9 (**Bk**)
28) 64 × 9 (**Fd**)
29) 137 × 9 (**Ek**)
30) 23 × 7 (**Dc**)
31) 26 × 4 (**Eb**)
32) 63 × 8 (**Aa**)
33) 83 × 7 (**Cj**)
34) 89 × 3 (**Dh**)
35) 714 ÷ 7 (**Ba**)
36) 2440 ÷ 8 (**Eℓ**)
37) 3525 ÷ 5 (**Cd**)
38) 12621 ÷ 3 (**Ai**)
39) 28622 ÷ 2 (**Fg**)
40) 3576 ÷ 2 (**De**)
41) 63718 ÷ 2 (**Af**)
42) 24 × 5 (**Cg**)
43) 86 × 5 (**Aℓ**)
44) 612 × 5 (**Ej**)
45) 874 × 5 (**Bg**)

SOLUTIONS

	a	**b**	**c**	**d**	**e**	**f**	**g**	**h**	**i**	**j**	**k**	**ℓ**
A	504	1004	305	257	18	31859	1370	67	4207	33	792	430
B	102	7846	789	901	41	433	4370	48	282	7	207	10842
C	6	24	411	705	1551	359	120	881	45	581	648	1072
D	45	112	161	1121	1788	612	73	267	432	74	1447	94
E	92	104	119	323	732	1642	505	36	481	3060	1233	305
F	227	44	667	576	1593	714	14311	885	951	47	830	33