



Comparing Fractions

It's often useful to be able to compare two or more fractions and decide which is the greatest. Because of the way fractions are written, it can be hard to tell just by looking at them. There are a number of methods you can use to answer problems like this.

Example 1: Which of these fractions is the largest? $\frac{4}{5}$ $\frac{7}{15}$ $\frac{2}{15}$ $\frac{7}{17}$ $\frac{29}{37}$

Solution: Even though we're asked to compare five different fractions, a good strategy is to compare only two at a time. We can then narrow down our choices.

The easiest way to start is to find two fractions with the same **denominator** (bottom number). The denominator tells us how many pieces one whole object has been divided into. For two fractions with the same denominator, the whole object has been cut into the same size pieces. The **numerator** (top number) tells us how many pieces we have.

Compare $\frac{7}{15}$ and $\frac{2}{15}$. The first fraction is seven pieces and the second is two pieces. $\frac{2}{15}$ must be smaller, so we can eliminate it. Cross it out in the question — it's wrong.

Another easy technique is to compare two fractions with the same numerator. Compare $\frac{7}{15}$ and $\frac{7}{17}$. Both fractions say we have seven pieces, but how big are the pieces? If I cut an object (like a pizza) into 3 pieces, the pieces will be larger than if I cut it into 30 pieces. Two slices of the first pizza is more than two slices of the second pizza.

Therefore, when the numerators are the same, the *smaller* denominator is the bigger fraction. $\frac{7}{15}$ must be larger than $\frac{7}{17}$, so we can cross $\frac{7}{17}$ off the list.

We can make two fractions fit into one of those techniques by finding a common denominator or a common *numerator*, and applying those two rules. Compare $\frac{4}{5}$ and $\frac{7}{15}$. They don't have a common number, but we can give them a common denominator by multiplying $\frac{4}{5}$ by $\frac{3}{3}$ to get $\frac{12}{15}$. $\frac{12}{15}$ is obviously bigger than $\frac{7}{15}$, so we can eliminate $\frac{7}{15}$ and keep $\frac{4}{5}$. (Similarly, we could easily compare $\frac{4}{13}$ and $\frac{16}{49}$ by rewriting $\frac{4}{13}$ as $4 \times \frac{4}{13 \times 4} = \frac{16}{52}$ — $\frac{16}{49}$ is larger than $\frac{16}{52}$ because its denominator is smaller.)

The last two fractions are $\frac{4}{5}$ and $\frac{29}{37}$. There doesn't seem to be any reasonable way to compare those; any attempt to make a common number will be messy. The last technique is great for cases like this.

Write the two fractions down side by side. Multiply the numerator of one fraction by the denominator of the other fraction, and write the answer beside the **numerator** you used. Do the same for the other side:

$$37 \times 4 = 148 \quad \frac{4}{5} \quad \frac{29}{37} \quad 29 \times 5 = 145$$

The fraction next to the larger answer is the larger fraction. In our question, the 4 in $\frac{4}{5}$ is part of the bigger answer, so $\frac{4}{5}$ is the largest fraction in the list.



One final technique is to compare your fractions to some “benchmark” number, like 0, $\frac{1}{2}$ or 1. For example, which is bigger, $\frac{21}{44}$ or $\frac{67}{120}$? As an estimate, those two numbers are both close to $\frac{1}{2}$. We can compare each fraction with $\frac{1}{2}$. $\frac{1}{2} = \frac{22}{44}$. We can see that $\frac{21}{44}$ is less than that because its denominator is less than 22. $\frac{1}{2} = \frac{60}{120}$, and $\frac{67}{120}$ is greater than that. Since one fraction is more than $\frac{1}{2}$ and the other is less, we know which fraction is greater: $\frac{67}{120}$. We can't do every question this way, but it can be a common-sense short cut to a hard question.

EXERCISES

A. Circle the greatest fraction in these lists.

- | | |
|---|--|
| 1) $\frac{16}{33}$ $\frac{19}{33}$ $\frac{7}{33}$ $\frac{23}{33}$ | 5) $\frac{12}{25}$ $\frac{3}{5}$ $\frac{18}{35}$ $\frac{4}{7}$ |
| 2) $\frac{8}{13}$ $\frac{8}{19}$ $\frac{8}{9}$ $\frac{8}{71}$ | 6) $\frac{3}{7}$ $\frac{6}{11}$ $\frac{29}{55}$ $\frac{7}{13}$ |
| 3) $\frac{13}{24}$ $\frac{7}{12}$ $\frac{1}{2}$ $\frac{3}{8}$ | 7) $\frac{12}{45}$ $\frac{35}{75}$ $\frac{21}{48}$ $\frac{49}{70}$ |
| 4) $\frac{30}{37}$ $\frac{3}{4}$ $\frac{10}{13}$ $\frac{6}{7}$ | 8) $\frac{108}{307}$ $\frac{23}{70}$ $\frac{4}{13}$ $\frac{31}{100}$ |

B. Circle the *least* (smallest) number in these lists.

- | | |
|--|---|
| 1) $\frac{4}{11}$ $\frac{8}{11}$ $\frac{6}{11}$ $\frac{3}{11}$ | 5) $\frac{3}{13}$ $\frac{1}{4}$ $\frac{11}{52}$ $\frac{3}{10}$ |
| 2) $\frac{5}{27}$ $\frac{15}{79}$ $\frac{5}{24}$ $\frac{15}{83}$ | 6) $\frac{4}{19}$ $\frac{3}{14}$ $\frac{1}{4}$ $\frac{12}{59}$ |
| 3) $\frac{12}{29}$ $\frac{36}{85}$ $\frac{2}{5}$ $\frac{9}{22}$ | 7) $\frac{6}{35}$ $\frac{7}{37}$ $\frac{8}{39}$ $\frac{9}{41}$ |
| 4) $\frac{37}{60}$ $\frac{23}{48}$ $\frac{53}{100}$ $\frac{11}{14}$
[Hint: How do these compare with $\frac{1}{2}$?] | 8) $-\frac{7}{12}$ $-\frac{3}{5}$ $\frac{1}{47}$ $-\frac{10}{17}$
[Hint: One of these is clearly the largest.] |

C. Put these fractions in order from least to greatest. (Find the least fraction, then the least out of those remaining....)

- | | |
|---|---|
| 1) $\frac{11}{14}$ $\frac{6}{7}$ $\frac{8}{7}$ $\frac{13}{14}$ | 4) $\frac{35}{72}$ $\frac{27}{50}$ $\frac{81}{160}$ $\frac{1}{2}$ |
| 2) $\frac{2}{7}$ $\frac{3}{10}$ $\frac{6}{19}$ $\frac{1}{4}$ | 5) $\frac{5}{8}$ $\frac{6}{11}$ $\frac{11}{15}$ $\frac{2}{3}$ |
| 3) $\frac{15}{19}$ $\frac{38}{29}$ $-\frac{19}{87}$ $\frac{15}{41}$ | 6) $\frac{13}{24}$ $\frac{10}{21}$ $\frac{5}{8}$ $\frac{4}{7}$ |

SOLUTIONS

- A: (1) $\frac{23}{33}$ (2) $\frac{8}{9}$ (3) $\frac{7}{12}$ (4) $\frac{6}{7}$ (5) $\frac{3}{5}$ (6) $\frac{6}{11}$ (7) $\frac{49}{70}$ (all these fractions reduce)
 (8) $\frac{108}{307}$ (all these fractions are close to $\frac{1}{3}$; only this one is greater than $\frac{1}{3}$)
- B: (1) $\frac{3}{11}$ (2) $\frac{15}{83}$ (3) $\frac{2}{5}$ (4) $\frac{23}{48}$ (5) $\frac{11}{52}$ (6) $\frac{12}{59}$ (7) $\frac{6}{35}$
 (8) $-\frac{3}{5}$: Since $\frac{1}{47}$ is the only positive number; it must be the largest, it can be eliminated first.
- C: (1) $\frac{11}{14} < \frac{6}{7} < \frac{13}{14} < \frac{8}{7}$ (2) $\frac{1}{4} < \frac{2}{7} < \frac{3}{10} < \frac{6}{19}$ (3) $-\frac{19}{87} < \frac{15}{41} < \frac{15}{19} < \frac{38}{29}$
 (4) $\frac{35}{72} < \frac{1}{2} < \frac{81}{160} < \frac{27}{50}$ (5) $\frac{6}{11} < \frac{5}{8} < \frac{2}{3} < \frac{11}{15}$ (6) $\frac{10}{21} < \frac{13}{24} < \frac{4}{7} < \frac{5}{8}$

