Everything About Algebra

One key skill that you will need in any math course or physical science course you do is **algebra**. The manipulation of formulas and equations is required in high school math, business math, calculus, physics and chemistry. This worksheet will give you the basic strategies for completing any algebra problem by outlining the two actions you take in solving algebra problems, and explaining the two reasons behind doing them.

TWO ACTIONS

Any step in solving an algebra problem can be classified as one of two general actions. There are rules for each action and examples demonstrating each action below.

ACTION 1) You may, at any time, replace a portion of an expression or equation with anything else that has the same value. If you replace a part of an equation with something that has a different value, you change the question, and you will get a different answer. Examples of this kind of step are:

Simplifying.

$$3x + 2x = 7 - 3$$
$$5x = 4$$

On each side of this equation, we've replaced a sum of two terms with the result of adding those terms. This is the easiest and most common use of Action #1.

Substitution.

$$y = 3x - 2$$
$$4x - 3y = 5$$

$$4x - 3(3x - 2) = 5$$

Here we have two equations to solve. We know the value of y in terms of x in the first equation. We replace the y in the second equation with an expression which is equal to y, and now we have one equation and one unknown, which is easier to solve.

Trig identities.

$$\csc\theta \tan\theta = \frac{1}{\sin\theta} \cdot \frac{\sin\theta}{\cos\theta}$$

Once you start doing trigonometry, you learn lots of identities — sets of expressions that are equal to each other. It is useful to use these to simplify equations that we would otherwise not be able to work on. (In this example, making these substitutions allows us to cancel sin θ, which makes this expression much simpler.)

Multiplying by one.

$$\frac{x}{\sqrt{3}} = \frac{x}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$$

Multiplying by one is a fancy way of saying that you can always multiply any number or term by a fraction that is equal to one (i.e., a fraction whose numerator and denominator are the same). Often this is a useful trick when dealing with fractions or rational expressions. (In this example, we chose a fraction equal to one that will let us rationalize the denominator.)

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ACTION 2) You may, at any time, <u>perform the same operation</u> (addition, subtraction, multiplication, division, . . .) to *all terms* on both sides of an equation. If you perform an operation on only part of an equation, you are changing the question and you will get a different answer.

Examples of this kind of step are:

Moving to the other side.

$$3x + 3 = 2x + 7$$
$$3x - 2x = 7 - 3$$

The "+ 3" on the left side has become "− 3" on the right side because we subtracted three from both sides. This "cancelled" the 3 on the left and introduced a 3 on the right. We did the same thing with the 2x on the right.

Removing fractions.

$$\frac{3}{x} + \frac{7}{x+2} = 5$$
$$[(x)(x+2)]\left(\frac{3}{x} + \frac{7}{x+2}\right) = 5[(x)(x + 2)]$$
$$3(x + 2) + 7x = 5x^2 + 10x$$

Even mathematicians don't like fractions. In a problem that contains them, we may want to multiply the entire equation by the lowest common denominator of all the fractions in the problem to remove the fractions.

Squaring both sides.

$$\sqrt{x+1} = 3$$
$$(\sqrt{x+1})^2 = 3^2$$
$$x + 1 = 9$$

This step is a combination of Action #1 and Action #2. We know that both expressions on the first line of this example are equal to each other. In the next line, we multiply each side by itself. Each side is multiplied by a different expression, but we know those expressions are equal, so it's okay.

The solution to every algebra problem is a series of steps that are examples of either the first or second kind of action. There is absolutely nothing else you can do. Let's look at the two reasons we do these actions, and then examine an algebra problem.

TWO REASONS

1) You do a step because it helps to <u>isolate</u> a variable whose value you want to know. In any question that says "solve", or that asks you to determine a missing number, you're trying to find a specific value for a variable that makes an equation or expression true.

For a linear expression, that means getting all terms with the unknown variable on one side, and everything else on the other. Then the goal is to get "x = ___" where the right side of the equation is a number. For some types of problems, we need to use a more complex strategy to find the solution. The simplest example is a higher-order polynomial, such as a quadratic equation. In a quadratic equation, the strategy is to get *everything* on one side, and 0 on the other, then try to factor. Once a polynomial is factored, we treat each binomial factor as a linear equation and isolate x to solve.

2) You do a step because it helps to <u>simplify</u> a complicated expression. We use this reason in the "solve" type of problem, but it is also the reason behind any action in a problem that says "evaluate" or that asks you to rewrite something in another form ("rationalize the denominator", "express in logarithmic form", "integrate", etc.). If there is a variable in a question like this, the variable stands for any number, and represents *all* numbers at once. $(x + 1)^2$ will be equal to $(x^2 + 2x + 1)$ no matter what number you plug in for x, not just for one particular number that you must determine.

Occasionally, we must make an expression more complicated that the one we started with in order to simplify it later on (see Example 2). Usually this is because we are solving a type of problem for which more direct methods don't work.

EXAMPLE PROBLEMS

Example 1: Solve: $(x)(x + 2) = 5(x - 2) - 8$

Solution: This problem is an equation, and we've been asked to solve it. Our strategy will be to simplify the expression first. Once that's done, we'll see that we have a quadratic expression. To isolate the unknown variable, we'll need to use the strategy described earlier: get everything to one side, zero on the other, and factor.

This solution has far more detail than you'd be expected to include in your homework or on a test, but it helps you to see the thinking behind every step of the solution.

PROBLEM			STEP	REASON
$(x)(x + 2)$	=	$5(x - 2) - 8$	original problem	
$x^2 + 2x$	=	$5(x - 2) - 8$	**replace:** expand	**simplify**
$x^2 + 2x$	=	$5x - 10 - 8$	**replace:** expand	**simplify**
$x^2 + 2x$	=	$5x - 18$	**replace:** subtract	**simplify**
$x^2 + 2x - 5x$	=	$5x - 18 - 5x$	**operate:** subtract 5x	**isolate:** quadratic strategy
$x^2 - 3x$	=	$5x - 18 - 5x$	**replace:** subtract	**simplify**
$x^2 - 3x$	=	-18	**replace:** subtract	**simplify**
$x^2 - 3x + 18$	=	$-18 + 18$	**operate:** add 18	**isolate:** quadratic strategy
$x^2 - 3x + 18$	=	0	**replace:** add	**simplify**
$(x - 6)(x + 3)$	=	0	**replace:** factor	**isolate:** quadratic strategy
$x - 6 = 0$	or	$x + 3 = 0$	**replace:** two cases	**isolate:** quadratic strategy
$x - 6 + 6 = 0 + 6$	or	$x + 3 - 3 = 0 - 3$	**operate**	**isolate**
$x = 0 + 6$	or	$x = 0 - 3$	**replace**	**simplify**
$x = 6$	or	$x = -3$	**replace**	**simplify**

Example 2: $(\sqrt{50} + \sqrt{48})(\sqrt{72} - \sqrt{12})$

Solution: In this problem, we have no unknown variable. Even though there's no instruction to tell us what to do, we can tell that this is an "evaluate" type of problem, rather than a "solve" type of problem. The reason for every step will be to simplify, and since there's no "other side", every step will be the replace action. This problem is fairly compact in the form it's given to you, but it will become more complicated-looking before it's simplified again.

PROBLEM		STEP	REASON
	$(\sqrt{50} + \sqrt{48})(\sqrt{72} - \sqrt{12})$	original problem	
=	$(\sqrt{25 \times 2} + \sqrt{16 \times 3})(\sqrt{36 \times 2} - \sqrt{4 \times 3})$	**replace:** factor	**simplify:** reduce radicals
=	$(\sqrt{25} \times \sqrt{2} + \sqrt{16} \times \sqrt{3}) \times (\sqrt{36} \times \sqrt{2} - \sqrt{4} \times \sqrt{3})$	**replace:** radical law	**simplify:** reduce radicals
=	$(5\sqrt{2} + 4\sqrt{3})(6\sqrt{2} - 2\sqrt{3})$	**replace:** square root	**simplify**
=	$5\sqrt{2} \cdot 6\sqrt{2} + 5\sqrt{2}(-2\sqrt{3}) + 4\sqrt{3} \cdot 6\sqrt{2} + 4\sqrt{3}(-2\sqrt{3})$	**replace:** FOIL	**simplify:** order of oper'ns
=	$30\sqrt{4} - 10\sqrt{6} + 24\sqrt{6} - 8\sqrt{9}$	**replace:** expand	**simplify**
=	$30 \cdot 2 - 10\sqrt{6} + 24\sqrt{6} - 8 \cdot 3$	**replace:** square root	**simplify**
=	$60 - 10\sqrt{6} + 24\sqrt{6} - 24$	**replace:** multiply	**simplify**
=	$36 - 10\sqrt{6} + 24\sqrt{6}$	**replace:** subtract	**simplify**
=	$36 + 14\sqrt{6}$	**replace:** subtract	**simplify**