Greatest Common Factor & Least Common Multiple

If you need to find the Greatest Common Factor, or the Least Common Multiple, of two numbers, it is useful first to find the prime factorizations of the numbers.

PRIME FACTORIZATION

Step 1: Divide the number by the smallest prime number which will divide it evenly. (Prime numbers are numbers that can only be divided evenly by themselves and 1. The first 10 prime numbers are: 2, 3, 5, 7, 11, 13, 17, 19, 23, and 29.)
Step 2: Divide the quotient by that prime again, if possible, or the next prime number that is a factor. Keep going until the quotient is prime.

Example: Find the prime factors of 36.
Solution:

$$\begin{array}{r|l} 2 & 36 \\ 2 & 18 \\ 3 & 9 \\ & 3 \end{array}$$

Therefore, the number 36 can be expressed as:
$36 = 2 \times 2 \times 3 \times 3 = 2^2 \times 3^2$.

GREATEST COMMON FACTOR

Step 1: Find the prime factorizations of each number.
Step 2: Write the product of the ***smallest*** power of each factor that appears in ***both*** factorizations.

Example: Find the GCF of 300 and 216.
Solution: $300 = 2 \times 2 \times 3 \times 5 \times 5 = 2^2 \times 3 \times 5^2$
$216 = 2 \times 2 \times 2 \times 3 \times 3 \times 3 = 2^3 \times 3^3$

The only prime factors of these numbers that appear in both are 2 and 3. The smallest power of 2 that appears is 2^2 and the smallest power of 3 is just 3. Therefore, the GCF is $2^2 \times 3 = 12$.

LEAST COMMON MULTIPLE

Method A: Using Prime Factorizations
Step 1: Find the prime factorizations of each number.
Step 2: Write the product of the ***largest*** power of each factor that appears in ***any*** factorization.

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Example: Find the LCM of 12 and 18.
Solution: $12 = 2 \times 2 \times 3 = 2^2 \times 3$
$18 = 2 \times 3 \times 3 = 2 \times 3^2$

The prime factors that appear in these numbers are 2 and 3. The largest power for either is squared, so the LCM is $2^2 \times 3^2 = 36$.

Method B: Using Multiples
Step 1: Write the set of multiples for each number.
Step 2: The first number that appears in all lists is the LCM.

Example: Find the LCM of 12 and 18.
Solution:

12: 12, 24, 36, 48, 60, …

18: 18, 36, 54, 72, 90, …

The LCM is 36.

EXERCISES

A. Find the prime factors of the following:

1) 24
2) 36
3) 45
4) 48
5) 60
6) 72
7) 81
8) 220

B. Find the greatest common factor of:

1) 20 and 25
2) 28 and 32
3) 21 and 54
4) 36 and 60
5) 72 and 144
6) 6, 8 and 12
7) 8, 20 and 44
8) 60, 100 and 200
9) 45, 75 and 225
10) 180, 450, 360 and 2250

C. Find the least common multiple of:

1) 12 and 16
2) 18 and 30
3) 4, 8 and 10
4) 3, 6 and 12
5) 24, 36 and 45
6) 45, 75 and 81
7) 25, 35 and 75
8) 10, 15 and 45
9) 3, 33 and 198
10) 18, 27, 60 and 90

SOLUTIONS

A. (1) $2^3 \times 3$ (2) $2^2 \times 3^2$ (3) $3^2 \times 5$ (4) $2^4 \times 3$ (5) $2^2 \times 3 \times 5$ (6) $2^3 \times 3^2$ (7) 3^4 (8) $2^2 \times 5 \times 11$

B. (1) 5 (2) 4 (3) 3 (4) 12 (5) 72 (6) 2 (7) 4 (8) 20 (9) 15 (10) 90

C. (1) 48 (2) 90 (3) 40 (4) 12 (5) 360 (6) 2025 (7) 525 (8) 90 (9) 198 (10) 540