Math Basics   Learning Centre

Ratio & Proportion

A **proportion** is an equality of two or more ratios:

$$\frac{a}{b} = \frac{c}{d} \quad [1]$$

In any proportion, the cross-products are equal:

$$ad = bc$$

Equation [1] could also be expressed using colon notation, as a **ratio**:

$$a : b : : c : d \ \text{ or } \ a : b = c : d$$

In this example, a and d are called the **extremes** and b and c are called the **means**. Using this terminology:

the product of the extremes $=$ the product of the means
$$ad = bc$$

Example 1:   Solve: $\dfrac{5}{25} = \dfrac{2}{x}$

Solution: Set up the cross products:
$$5 \cdot x = 25 \cdot 2$$
Simplify:
$$5x = 50$$
Solve for x by dividing both sides by the coefficient of x:
$$5x \div 5 = 50 \div 5$$
$$x = 10$$

Example 2:   Solve: $\dfrac{2}{3} = \dfrac{\frac{1}{2}}{x}$

Solution: Set up the cross products:
$$2 \cdot x = 3 \ \cdot \tfrac{1}{2}$$
Simplify:
$$2x = \tfrac{3}{2}$$
Solve for x. We could do this by dividing both sides by the coefficient of x, but since we have a fraction it might be easier to multiply both sides by the reciprocal of the coefficient (which amounts to the same thing). The reciprocal of 2 is ½, so:
$$2x \times \tfrac{1}{2} = \tfrac{3}{2} \times \tfrac{1}{2}$$
$$x = \tfrac{3}{4}$$

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Example 3: If it costs 31 cents to cut 4 keys, how much would it cost to cut 12 keys?

Solution: Form a proportion for the problem. The units should match on each side of the equal sign (numerators should have the same units and so should the denominators):

$$\frac{31 \text{ cents}}{4 \text{ keys}} = \frac{x \text{ cents}}{12 \text{ keys}}$$

Set up the cross-products:

$$31 \cdot 12 = 4 \cdot x$$

Multiplying 31 × 12 is hard. It is easier to divide by the coefficient now to make the calculations simpler:

$$(31 \cdot 12) \div 4 = 4x \div 4$$
$$31 \cdot 3 = x$$
$$93 = x$$

Write your answer to the question:

It would cost 93¢ to cut 12 keys.

Example 4: In Mark's new job, he is paid $1700 every two weeks. What is Mark's annual salary?

Solution: Form a proportion for the problem. The units should match on each side of the equal sign, so we have to convert between years and weeks to solve the problem:

$$\frac{\$1700}{2 \text{ weeks}} = \frac{\$x}{1 \text{ year}}$$

$$\frac{\$1700}{2 \text{ weeks}} = \frac{\$x}{52 \text{ weeks}}$$

Set up the cross-products:

$$\$1700 \cdot 52 = 2 \cdot x$$

Once again, it will be easier to divide by the coefficient now rather than later:

$$(\$1700 \cdot 52) \div 2 = 2x \div 2$$
$$\$1700 \cdot 26 = x$$
$$\$44{,}200 = x$$

Write your answer to the question:

Mark's annual salary is $44,200.

EXERCISES

A. Solve for x.

1) $\dfrac{7}{24} = \dfrac{x}{48}$	12) $\dfrac{24}{72} = \dfrac{x}{9}$
2) $\dfrac{3}{4} = \dfrac{x}{60}$	13) $\dfrac{8}{x} = \dfrac{3}{9}$
3) $\dfrac{30}{5} = \dfrac{12}{x}$	14) $7 : 3.5 = 21 : x$
4) $\dfrac{10}{5} = \dfrac{x}{7}$	15) $15 : 25 = x : 100$
5) $\dfrac{15}{x} = \dfrac{3}{6}$	16) $x : 4 : : 255 : 60$
6) $\dfrac{15}{x} = \dfrac{12}{4}$	17) $0.6 : x = 0.78 : 0.325$
7) $\dfrac{15}{x} = \dfrac{20}{4}$	18) $0.5 : 0.75 = x : 1$
8) $\dfrac{5}{20} = \dfrac{x}{32}$	19) $2 : 3 = \frac{1}{2} : x$
9) $\dfrac{4}{8} = \dfrac{8}{x}$	20) $\frac{7}{8} : \frac{3}{16} = x : \frac{5}{8}$
10) $\dfrac{7}{52} = \dfrac{7}{x}$	21) $20 : 2.5 = x : \frac{1}{2}$
11) $\dfrac{50}{75} = \dfrac{x}{\frac{1}{2}}$	22) $0.1 : 0.001 = 0.01 : x$

B. Use a proportion to solve the following problems:

1) A supply of 500 halibut liver capsules sells for $4.80. What would be the price for 125 capsules?

2) In the Tofu Tasting Club, there are 45 women. The ratio of men to women is 3 to 5. How many men are there in the club?

3) If you had the uncanny ability of being able to crack open coconuts with your head at the rate of 4 coconuts every 9 minutes, how many coconuts could you crack open in 45 minutes?

4) A speedboat passes a race checkpoint 52.5 miles past the start of the course 2 hours after the race started. If the entire course is 210 miles long, how much time would you expect the speedboat to take to finish?

5) A manufacturing plant can make 750 microwave ovens in 9 days. How large an order for microwave ovens can the plant fill in 33 days?

6) A woman bought 180 acres of bog in Richmond for $51,000 and promptly sold 30 acres of it to you at cost. How much did she receive for your plot of land?

7) If you were to be paid $8.50 for two hours of work, how much should you receive in total for working a shift that is three hours longer?

8) If sound travels 825 metres in 2.5 seconds, how long will it take for the explosive sound of a cannon firing to travel 3.3 km? (Remember: 1 km = 1000 m)

9) 500 mL of liquid hand soap sells for $1.58. How much would 1.5 L of liquid hand soap cost? (Remember: 1 L = 1000 mL)

10) A special camera can take 500 pictures in 33⅓ seconds. How many pictures can the camera take in 7 minutes of constant use?

SOLUTIONS
A. (1) 14   (2) 45   (3) 2   (4) 14   (5) 30   (6) 5   (7) 3   (8) 8   (9) 16   (10) 52   (11) ⅓
   (12) 3   (13) 24   (14) 10.5   (15) 60   (16) 17   (17) .25   (18) $0.\overline{6}$ (19) ¾   (20) $^{35}\!/_{12}$
   (21) 4   (22) 0.000 1
B. (1) $1.20   (2) 27 men   (3) 20 coconuts and 1 splitting headache   (4) 8 hours
   (5) 2750 microwave ovens   (6) $8500   (7) $21.25 for 5 hours   (8) 10 seconds
   (9) $4.74   (10) 6300 pictures