Making Abstract Problems Concrete

The one thing that most math students have trouble with is word problems. This can be an obstacle once you leave school because most real-world problems that we encounter involving math happen in the style of word problems: we're in a situation, we have access to a bunch of numerical data that may or may not help us to solve the problem, and we have to figure out *how* to find the solution and then do it.

Here's a secret. You may have thought to yourself, "When am I ever going to need this after I'm done with school?" The answer is one of these three reasons:

- Math concepts are frequently used even in "non-math" jobs.
- Your teachers are getting you prepared for a later course, whether it's economics for your business degree, statistics for your nursing degree, or a future math course. You'll need what you're doing now to understand that future material.
- You're not going to need this. But the real point is sharpening your problem-solving skills, and you'll use those every day no matter what job you do.

The third one is the most important one. An employee who can find solutions to problems independently is much more valuable than one who has to wait to be told what to do all the time.

The biggest step in solving word problems is turning the problem into an equation. Once that's done properly, it's much easier to plug in the numbers and solve the problem. People do find it easier to work with numbers rather than variables. Here's a way you can use that to your advantage if you're stuck turning a process into an equation.

An equation is meant to be a template for solving all problems of a certain type, no matter what numbers you use. This means any number will work in the equation. If you pick a number and watch how you perform a calculation, you can turn it into a formula to solve problems. Here's an example to demonstrate what that means:

Example 1: Monica is renting a car for the weekend. The rental company charges a fee of $25 for the rental plus $0.75 for every kilometre the car is driven. Monica can only afford to spend $120 on the rental. How far can she drive the car?

[This is a classic word problem. If we were told the distance she drives over the weekend, we'd be able to calculate the cost — the question tells us how. This word problem, however, is going backwards; we've been told the cost and we need to find the distance. The only way to do this without guessing is to find the formula and work backwards. If you would get stuck at the point, use a concrete value to see how the calculation is performed, and build a formula from that.]

Authored by Darren Rigby

Solution: If it's not clear what formula to use, we can use any value, say 20 km, and see how we would calculate the cost. (20 km isn't the answer, but that doesn't matter.)

We have to pay the flat fee:	\$25
To that, we add \$0.75 for every kilometre we drive, so that's 0.75 × 20, or \$15	\$25 + 15 = \$40

We've solved the problem once, so now we can analyse what we did. The number we used to start the problem is the number of kilometres. We took that number and multiplied by the rate per kilometre, \$0.75. Last, we added the flat fee to that. If we call the number of kilometres x:

We take that number and multiply by the rate per kilometre:	$0.75x$
We add the flat fee to that:	$0.75x + 25$

This works because it doesn't matter how many kilometres we drive, we calculate the cost the same way. We have a formula: cost = 0.75x + 25. Now we can plug in Monica's maximum cost and solve.

$$\begin{aligned} 120 &= 0.75x + 25 \\ 120 - 25 &= 0.75x + 25 - 25 \\ 95 &= 0.75x \\ \frac{95}{0.75} &= \frac{0.75x}{0.75} \\ 126.6667\ldots &= x \end{aligned}$$

Monica can drive 126 full kilometres without going over budget.

Some problems are really abstract. They can be solved the same way.

Example 2: If I add two consecutive whole numbers, and double the result, the answer is two less than if I had multiplied the numbers. What are the numbers?

Solution: Again, let's pick two consecutive numbers and see what calculations we perform. Let's choose a small number to make it easy: 2. The other whole number would then be 3, which is one more than 2 (consecutive).

We add them:	$2 + 3 = 5$
Double the result.	$5 \times 2 = 10$
Now multiply the original numbers:	$2 \times 3 = 6$
Subtract 2 and see if they're the same:	$6 - 2 = 4 \neq 10$

They're not the same, but now we know what we're doing. If the small whole number is x, here's what we would do:

Add the next whole number, which is x + 1:	$x + (x + 1) = 2x + 1$
Double the result:	$2 \times (2x + 1) = 4x + 2$
Now multiply the original numbers:	$x(x + 1) = x^2 + x$
Subtract 2 from that, and they should be the same:	$x^2 + x - 2 = 4x + 2$
	$x^2 - 3x - 4 = 0$

Now we have an equation. We can solve it by factoring:

$$x^2 - 3x - 4 = 0$$
$$(x - 4)(x + 1) = 0$$
$$x = 4 \text{ or } x = -1$$

The answers must be positive since the question asks for whole numbers, so the smaller number is 4. The numbers are 4 and 5.

EXERCISES

A. Trent is an electronics salesperson who works on commission. He figures he can buy a motorcycle and afford the payments if he maintains a weekly salary of at least $1350. He earns $425 each week plus 30% of the amount he sells.

1) If Trent sells $1000 of electronics in a week, how much is his take-home pay?

2) Starting with the amount Trent sells, it's a two-step process to calculate his pay for the week. What are the two steps? Describe the steps using numbers.

3) Write the formula for this problem.

4) Determine how much Trent has to sell per week to afford his motorcycle payments.

B. Amrit makes an investment at 7.5% interest, compounded annually. After a year the investment is worth $1236.25. How much was the initial investment?

1) If you invested $100 at that interest rate, how much would it be worth in a year?

2) Starting with the investment, what steps are needed to calculate the value after a year? Describe the steps using numbers.

3) Write the equation for this problem.

4) Determine the amount of Amrit's investment.

C. If I take a particular number x, square it, and add 3, I get the same answer as if I'd taken x, multiplied it by the next integer higher than x and subtracted 4. What's the number?

1) If you use 2 for x, what value do you get for the first calculation? For the second?

2) If you use x itself, what value do you get for the first calculation? For the second?

3) Write the equation for this problem.

4) Determine the value of x.

SOLUTIONS

A. (1) $725 (2) Multiply the amount he sells by 0.30. Add $425. (3) $0.30x + 425$ (4) $3083.33

B. (1) $107.50 (2) Multiply the investment by 0.075. Add the investment. [or "Multiply the investment by 1.075."] (3) $1236.25 = x + 0.075x$ [or $1236.25 = 1.075x$] (4) $1150

C. (1) 7, 2 (2) $x^2 + 3$, $x(x + 1) - 4$ (3) $x^2 + 3 = x(x + 1) - 4$ (4) 7