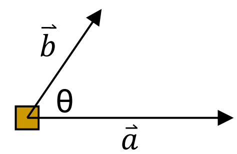
Vectors

ADDING VECTORS

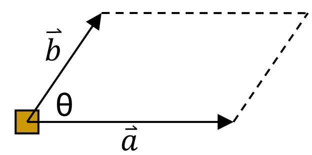
Parallelogram Method

The parallelogram method is used to add two vectors acting at an angle θ with respect to each other.

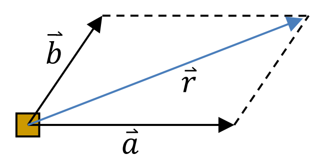
Consider two vectors, $\vec{a}$ and $\vec{b}$, acting on the same object:



From the tip of vector $\vec{a}$, draw a line equal in length to $\vec{b}$, and parallel to $\vec{b}$. From the tip of vector $\vec{b}$, draw a line equal in length to $\vec{a}$ and parallel to $\vec{a}$.

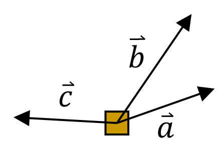


Draw the diagonal from the object to the opposite corner. This vector is called the **resultant**, and it is the vector of the sum of $\vec{a}$ and $\vec{b}$. The resultant, acting alone, produces the same effect as its component vectors acting together.

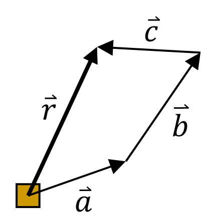


Graphical Addition

If the graphical representations of vector are drawn with *high* precision, the resultant can be measured with considerable accuracy, regardless of how many vectors are being added. This is where the tip-to-tail method is used. Consider the three vectors acting on the object at right.



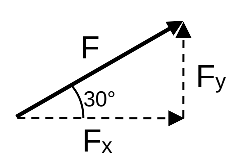
Arrange all the vectors from tip to tail. The resultant is the vector from the tail of the first vector to the tip of the last vector.



RESOLVING VECTORS INTO COMPONENTS

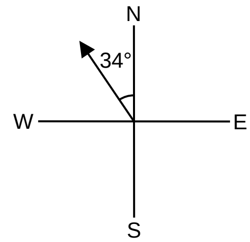
Often it is useful to break down (resolve) a vector into components. Usually we break a vector into horizontal and vertical components, but in the case of movement on a ramp, we might break a vector into components parallel to the surface of the ramp and perpendicular to the surface.

Consider a 10.0-N force, F, acting at a 30° angle to the horizontal, as in the diagram to the right. We can resolve this vector into horizontal and vertical components. The horizontal component, $F_x = F \cos 30° = 8.66$ N, and the vertical component, $F_y = F \sin 30° = 5.00$ N



Authored by Gordon Wong

ANGULAR REPRESENTATION

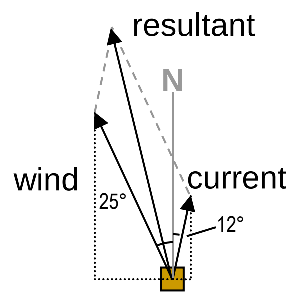


For real-life applications, angles are usually represented using compass notation. Vector directions are usually defined by an acute angle measured from north or south. There are two ways of writing the angle in the diagram at the right. **N 34° W** means the angle spans 34° to the west from due north, and **34° W of N** means the same thing.

(*More help is available in the Math 093 worksheet Directions of Travel.*)

Example 1: Two forces are acting on a sailboat: a current is exerting a force of 1500 N at N 12° E, and the wind is pushing the sail with a force of 3200 N at N 25° W. What is the net force (expressed as the vector sum of the current and the wind) on the sailboat?

Solution: To solve this problem, we must break the two forces down into their individual components, then add the components to determine the resultant force.



We make a diagram and create a right-angled triangle for each vector we are adding, breaking them down into their north-south and east-west components. Since both forces are pushing northwards, they are working together and the north-south components are added. The forces are pushing in opposite directions in the east-west direction, so they are subtracted. If we define east to be the positive direction along this axis, then the wind is pushing in a negative direction, so its east-west component will be subtracted. Since our diagram suggests that this component will be larger than the corresponding component from the current, we expect to get a negative answer; the sailboat should be going vaguely northwest. We can track this information with a table.

	north-south (y)	east-west (x)
current	1500 cos 12° ≈ 1467.221 N	1500 sin 12° ≈ 311.868 N
wind	3200 cos 25° ≈ 2900.185 N	3200 sin 25° ≈ (−)1352.378 N
total	4367.406 N	−1040.510 N

We get the size of the resultant by using the Pythagorean Theorem:

$||r|| = \sqrt{(4367.406)^2 + (1040.510)^2} = 4489.643\ldots \approx 4500$ N

We can get the angle of the resultant net force with trigonometry. We'll want to express our answer using the same system as was used in the question, so we'll need to know the angle that the resultant makes with the due North line. The north-south component makes up the adjacent side, and the east-west component makes up the opposite side. (We don't need to use the negative part of the east-west component, since that tells us direction, not distance.) We use tangent:

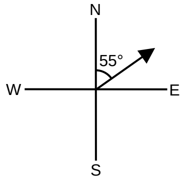
$\tan^{-1} \frac{1040.510}{4367.406} \approx 13.4°$

Therefore the resultant is 4500 N, N 13° W.

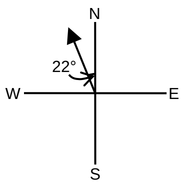
EXERCISES

A. Determine the direction of the following vectors using compass notation:

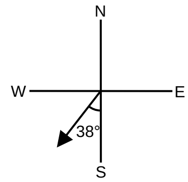
1)



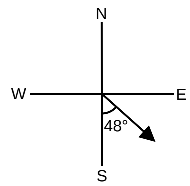
2)



3)



4)



B. An object is displaced 8.0 m east and 15 m north. What are the magnitude and direction of the resultant displacement?

C. From his house, a confused student travelled 4 blocks east, 3 blocks north, 3 blocks west and 6 blocks south. How far, and in what direction is he from home?

D. A vector has magnitude of 50.0 N and makes an angle of 36° with the horizontal. Determine the horizontal and vertical components of the vector.

E. A 3-N force acts on an object in the positive x-direction, and a 5-N force acts on the same object at an angle of 60° above the positive x-axis. Determine the magnitude and direction of the resultant force.

F. The current in a river flows south at a rate of 5.00 $^{km}/_{h}$. A woman rows a boat west at a rate of 3.00 $^{km}/_{h}$. What is the speed and direction of the boat relative to the ground?

G. A helium balloon will rise at a rate of 2.0 ${}^{m}\!/_{s}$ in still air. If a helium balloon is released in a 15 ${}^{km}\!/_{h}$ wind, what is the resultant speed of the balloon, and at what angle of elevation does it move?

H. An airplane flies 250 km in the direction 55.0° E of S. Find:
   1) the easterly component of the flight   2) the southerly component of the flight

I. Find the magnitude and direction of the resultant of two displacements: 6.00 km east and 7.00 km N 65° E.

J. A helicopter travels 15 km at 50° W of N; then flies 22 km at 42° E of N. Find the displacement and direction of the helicopter from the starting point.

SOLUTIONS

A. (1) 55° E of N (or N 55° E)   (2) 22° W of N (or…)   (3) 38° W of S   (4) 48° E of S
B. 17 m, 28.1° E of N   C. 3.2 blocks, 18.4° E of S   D. horiz: 40.5 N, vert: 29.4 N
E. 7 N, 38.2° above the positive x-axis   F. 5.83 ${}^{km}\!/_{h}$, 31.0° W of S
G. 4.6 ${}^{m}\!/_{s}$, 26° above the horizontal   H. (1) 205 km   (2) 143 km
I. 12.7 km, 76.5° E of N   J. 26.2 km, 7.1° E of N