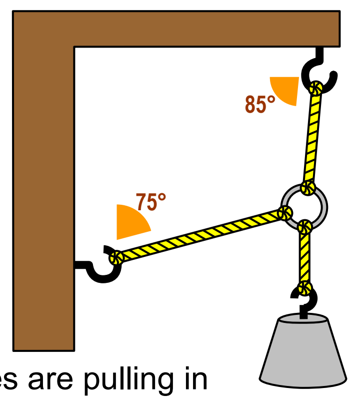
Equilibrium

When an object is perfectly still, then we know that there is no net force acting on the object. This is *not* the same as saying that there are no forces acting on the object. Usually, there is at least gravity and some counterbalancing force (usually a normal force or tension) at work. Equilibrium problems in physics use this fact to determine information about the individual forces in play in a situation. To solve these types of problems, we split the forces into horizontal and vertical components, and we may also have to consider the torque on an object.

The standard formula for force problems is: $\Sigma F = m \cdot a$. Since in equilibrium problems nothing is moving, $a = 0$ and this becomes $\Sigma F = 0$. We specify the directions to get:

$$\Sigma F_x = 0 \qquad \Sigma F_y = 0 \qquad \Sigma \tau = 0$$

Example 1: The ring in the diagram at the right is supported by two ropes under tension and the ring supports a load. The tension in the rope tied to the wall is 25 N. What is the weight of the load?



Solution: Since there's nothing turning in the diagram, there is no torque involved in the solution of the problem. We can use the first two equations for forces in the x- and y-directions to solve the problem. Call the tension in the ropes tied to the wall and ceiling T_1 and T_2, and the load W.

The ring has two horizontal forces acting on it: the horizontal components of the tensions in the supporting ropes. Since the ropes are pulling in opposite directions, and since everything is in equilibrium, their horizontal components must be equal. (We can't say the same about the vertical components — with the ceiling rope's tension and the load unknown, we don't have enough information to resolve the problem.) We'll start with the horizontal part.

Some students try to memorize that you use cos to find the horizontal component of a vector and sin to find the vertical component. This is only true if the angle is given relative to a horizontal line. If we tried to use this technique on the 75° angle on the wall rope we'd get an incorrect answer. The rope tied to the wall has an overall tension of 25 N and the rope itself is mostly horizontal. As a sanity check, the horizontal component of that tension should be close to 25 N and the vertical component should be much less.

$\Sigma F_x = 0$ $T_{1x} - T_{2x} = 0$ $T_{1x} = T_{2x}$ $25\text{ N} \cdot \sin 75° = T_2 \cdot \cos 85°$ $24.15 = 0.087\, T_2$ $T_2 = 24.15 \div 0.087 = 277.06890\ldots\text{ N}$ *(continued in next column)*	$\Sigma F_y = 0$ $T_{2y} - T_{1y} - W = 0$ $T_{2y} - T_{1y} = W$ $277.07 \sin 85° - 25 \cos 75° = W$ $W = 269.544\ldots \approx 270\text{ N}$

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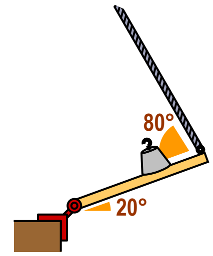
Example 2: A drawbridge (uniform, weighing 70. N) hangs from a cable and makes an angle of 20.° with the horizontal. There's a 240-N weight ¾ of the way to its free end. The cable currently makes an 80.° angle with the drawbridge. Find the tension in the cable and the total force exerted by the drawbridge's hinge.



Solution: Since the drawbridge pivots on its hinge, there is torque on it as well as traditional linear forces. If we look at torque first, and we use the hinge as our axis of rotation, then the forces acting on the hinge all have a distance of 0 and they won't come into the equation. The only unknown will be the tension in the cable. The drawbridge itself is "uniform", which means its density is the same everywhere, so its weight (mass) is distributed evenly throughout. This means its centre of gravity is exactly halfway down its length. (We don't need to know how long the drawbridge is; in equilibrium problems length will cancel out, leaving us looking at just fractions of the length.) Consider the forces holding the drawbridge up to be positive:

$\Sigma\, \tau = 0$

$\tau_{cable} - \tau_{drawbridge} - \tau_{weight} = 0$

$\tau_{cable} = \tau_{drawbridge} + \tau_{weight}$

$T \cdot \sin 80° \cdot L = 70\text{ N} \cdot \sin 70° \cdot \tfrac{1}{2}L + 240\text{ N} \cdot \sin 70° \cdot \tfrac{3}{4}L$

$0.985\, T = 32.889 + 169.145$

$T = 202.034 \div 0.985 = 205.151 \approx 210\text{ N}$

Now that we have the tension, we can find the components of the force acting on the hinge. Only the cable's tension acts on the drawbridge (and therefore the hinge) in the x-direction. The weight and the drawbridge are partially held up by the cable in the y-direction; the rest of the force is provided by the hinge.

We need to know the angle the cable makes with either the horizontal or the vertical to finish this problem. If we draw a horizontal line through the end of the drawbridge, cutting through the vertex of the 80° angle, the part that's below the line is 20°. (The drawbridge makes a 20° angle with the horizontal no matter which end we measure from.) This means the part above it must be 60°, so the cable is 60° from the horizontal.

$\Sigma\, F_x = 0$

$T_x - F_{hinge,x} = 0$

$T_x = F_{hinge,x}$

$F_{hinge,x} = 205.151 \cdot \cos 60° = 102.575\ldots$

$\Sigma\, F_y = 0$

$F_{hinge,y} + T_y - W_{drawbridge} - W_{weight} = 0$

$F_{hinge,y} = W_{drawbridge} + W_{weight} - T_y$

$F_{hinge,y} = 70 + 240 - 205.151 \cdot \sin 60°$

$= 132.334\ldots$

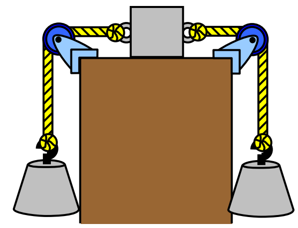
$\theta_F = \tan^{-1}(132.334 / 102.575)$

$= 52.220\ldots \approx 52°$

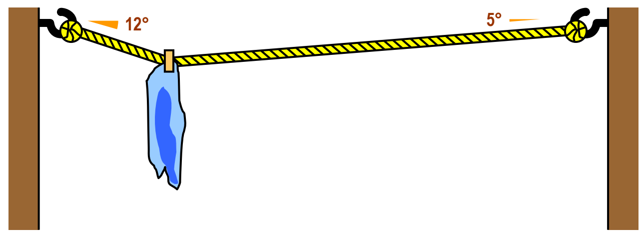
$\|\, F_{hinge} \,\| = \sqrt{132.334^2 + 102.575^2}$

$= 167.434 \approx 170\text{ N}$

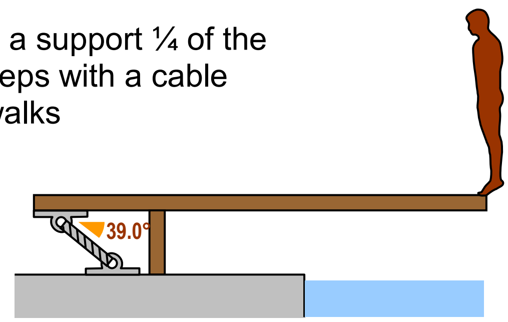
EXERCISES



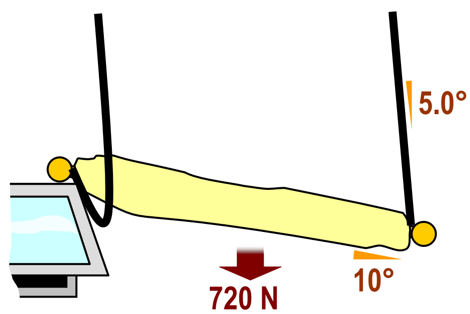
A. A block rests on a table, attached to ropes that pass through pulleys. Suspended from the ropes are two weights. The heavier of these two weights is 45 N. The block on the table is just on the verge of sliding, and its weight is 36 N. The coefficient of friction between the block and the table is 0.16. How heavy is the other weight?



B. A wet shirt is pinned on a clothesline, causing the line to sag unevenly. The shirt weighs 18 N. What is the tension in each section of the rope? [*Hint: the answers aren't the same. Because the shirt isn't free to slide on the clothesline, this system won't balance itself out.*]



C. A diving board is constructed by resting a plank on a support ¼ of the way along the plank, and securing the end near the steps with a cable stay. The plank is uniform and weighs 655 N. A man walks out to the end of the diving board, and as he's standing at the end, preparing to dive, the cable experiences 3520 N of tension. How much does the man weigh? [*Hint: There are torques about the location of the support. Use this point as an axis of rotation.*]



D. A rod with a banner wrapped around it is being lowered down the side of a building on two cables. One end of the banner gets snagged on an open window while the other side continues to be lowered. The rod and banner have uniform density along their length and together they weigh 720 N. The crew lowering the banner have noticed the problem and stopped the procedure in the position shown. Find the force (magnitude and direction) exerted on the window frame by the rod and the tension in the cable.

SOLUTIONS

A. 39 N

B. left side: 61 N; right side: 60 N

C. 520. N

D. tension = 3.7×10^2 N; force from banner = 3.6×10^2, 85° below the horizontal