Newton's Second Law

Newton's Second Law of Motion relates the ideas of force, mass and acceleration. It can be expressed with a simple formula:

$$\text{net force} = \text{mass} \times \text{net acceleration}$$
$$F_{net} = m \cdot a$$

This formula will be the starting point for many problems, so it's important to know exactly what it means and how to use it.

There are often multiple forces acting on an object at the same time. The **net force** is the vector sum of all those forces. The **net acceleration** of an object is the instantaneous acceleration of the object at the moment we're considering. If the object is moving in a straight line, then acceleration follows that line.

We solve a Newton's Second Law question by considering each object affected by forces individually. We draw a **free-body diagram** for the object, sketching the forces involved, so we know what to add.

A CATALOGUE OF FORCES

Gravity. Gravity acts on all objects near the Earth's surface. (If the problem doesn't explicitly say we're in space, there's gravity!) The force of gravity is equal to the object's weight: **W** or F_g = mg, where g = 9.81 $^m/_{s^2}$ is the acceleration due to gravity. You might be told to round to 10 $^m/_{s^2}$. Gravity always points *straight down*, never to the side.

Normal force. An object experiences a normal force (F_N) when it's in contact with a surface which supports the object, such as a book on a table. The table exerts a force on the book to support it and stop it from falling. If we put too heavy an object on the table, then the table's structure can't provide enough normal force, and the table breaks. Normal forces are reactive rather than proactive: only enough is provided to balance any existing forces from other sources. Normal forces are always *perpendicular to the surface that provides the force*. For vectors, **normal** means "perpendicular".

Tension. Tension (F_T or **T**) is similar to normal force, but it's provided by ropes and strings. Ropes exert tension to keep from breaking. Tension is reactive also; it can't be calculated directly. Tension's direction points *inwards along any section of rope*, in two directions at once, and (assuming massless pulleys) the tension in a rope is the same everywhere. When multiple sections of the same rope exert a force on an object, *each section* exerts a force; the forces are cumulative.
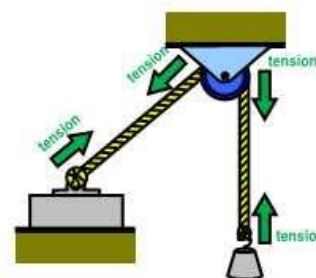
Friction. When two objects are in contact with each other, and an external force is applied to create motion, the objects rub together. Friction is the force that resists motion because of rubbing. The magnitude of friction is determined partly by the

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materials the objects are made of, expressed as a **coefficient of kinetic friction (μ_k)** or, if all motion is prevented by friction, the **coefficient of static friction (μ_s)**. Friction is also determined by the normal force between them — a lighter object will slide more easily than a heavy object. The formula for friction is $F_f = \mu \cdot F_N$. Friction is always a counterforce; it always points *opposite the direction of motion*.

Once have your free-body diagram, write Newton's Second Law with "F_{net}" replaced by the sum of the forces. (Any force that acts perpendicular to the direction of motion isn't included, and must be exactly opposed by some other force — this fact is often an important clue to solving the problem!)

Determining whether forces are positive or negative depends on what sign convention you use. A **sign convention** is a rule that defines what "positive" means in a problem. Sometimes we use the x- and y-axes as a sign convention: up is positive and down is negative; right is positive and left is negative. If you use this, you must particularly remember that any motion caused by gravity must be negative. A more intuitive sign convention is to declare that the direction of an object's motion is positive, no matter what direction that is. Then any force that opposes the motion is negative.

Each object should give you one equation, and the equations from all the objects should form a system of equations.

Example 1: Two weights, one 3 kg and one 7 kg, are suspended by a rope that passes over a massless pulley. The objects are released from rest. What is the tension in the rope and what is the acceleration of each weight?

Solution: We'll use Newton's Second Law to solve this problem. We start with a free-body diagram of each object.
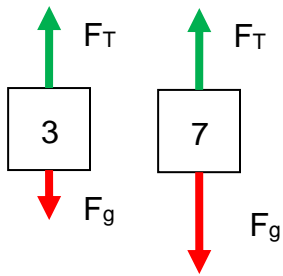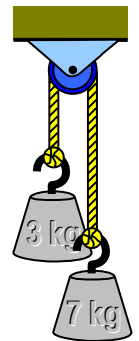
Both weights are hanging from rope, so each experiences a tension force. It's the same rope, so the tension forces are equal. The weights are of course also affected by gravity. We would expect the 7-kg weight to fall and the 3-kg weight to rise. This suggests that the force of gravity on the 7-kg weight is stronger than the tension force holding it up, and that the tension on the 3-kg weight is stronger than gravity. (We also show these forces' relative magnitudes on our diagram. The tension forces are the same, and the force of gravity is longer or shorter.) We also expect the accelerations to be the same for each object, not counting direction: if the 7-kg weight falls 1 m, then the 3-kg weight rises 1 m. (In more complex pulley systems this may not be true.)

We'll use the sign convention: up is positive and down is negative. We write an equation for each weight based on Newton's Second Law. Because we're examining each object separately, we don't use the mass of the whole system. Note that since one object is rising and one falling, the accelerations for the two objects aren't *quite* the same.

For the 3-kg weight:
$$F_{net} = m_1 \cdot a$$
$$F_T + F_g = (3\ \text{kg})(a)$$
$$F_T + m_1 \cdot (-g) = 3a$$
$$F_T + (3\ \text{kg})(-9.81\ \text{m/s}^2) = 3a$$

For the 7-kg weight:
$$F_{net} = m_2 \cdot (-a)$$
$$F_T + F_g = (7\ \text{kg})(-a)$$
$$F_T + m_2 \cdot (-g) = -7a$$
$$F_T + (7\ \text{kg})(-9.81\ \text{m/s}^2) = -7a$$

$$F_T - 29.43 = 3a \qquad\qquad F_T - 68.67 = -7a$$

We now have two equations and two unknowns: F_T and a. We can solve the system by substitution.

$$F_T = 3a + 29.43$$

$$(3a + 29.43) - 68.67 = -7a$$
$$3a - 39.24 = -7a$$
$$10a = 39.24$$
$$a = 3.924 \text{ m/s}^2$$

$$F_T = 3(3.924) + 29.43$$
$$= 41.202 \text{ N}$$

∴ The tension in the rope is approximately 40 N, the acceleration of the 7-kg weight is −4 m/s² and the acceleration of the 3-kg weight is 4 m/s².

Example 2:   Determine the rope's tension and acceleration for the system in the diagram. The pulley is massless, the 9.0-kg block and the ramp have a μk of 0.25, and the ramp is at 37°. The hanging weight is 3.0 kg.
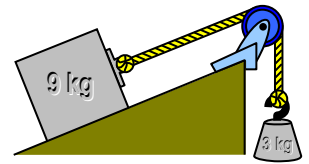
Solution: We'll follow the same procedure as before, but the ramp makes things a little more complex.
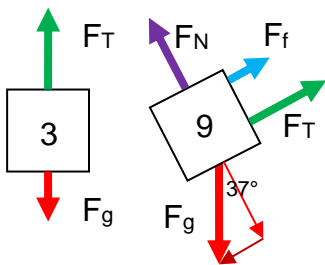
The free-body diagram for the hanging weight is exactly the same as the last problem. The block on the ramp has a couple of extra forces: a normal force and a friction force. We'll also need to break gravity into its components, since gravity is neither parallel nor perpendicular to the direction of motion. As we have it in the diagram, we've assumed that the heavier 9-kg block will slide down the ramp, raising the 3-kg block. Can we be sure about this? The 3-kg block won't be impeded by friction and a 37° ramp isn't that steep. It's hard to know which way the objects will move. The good news is even if we guess wrong, the problem will tell us. We'll get a negative acceleration, which means the objects are moving in the opposite direction from the way we set the problem up.

This time, we'll use the convention that forward motion is positive. You can think of this as which forces are "winning" and "losing". For the block (we assume) gravity wins. For the weight, tension wins. All the other forces in the problem lose. The winning forces get added; the losing forces get subtracted, and we don't have to worry about making any of the numbers negative on top of that. They'll already have the proper sign.

For the weight:
$$F_T - F_g = m_1 \cdot a$$
$$F_T - m_1 \cdot g = m_1 \cdot a$$
$$F_T - (3)(9.81) = 3a$$
$$F_T = 3a + 29.43$$

For the block:
$$F_{g\parallel} - F_f - F_T = m_2 \cdot a$$
$$m_2 \cdot g \sin 37° - \mu_k \cdot F_N - F_T = m_2 \cdot a$$
$$(9)(9.81)(0.60) - (0.25)(m_2\, g \cos 37°) - F_T = 9a$$
$$52.974 - (0.25)(9)(9.81)(0.40) - F_T = 9a$$
$$52.974 - 8.829 - (3a + 29.43) = 9a$$
$$14.715 = 12a$$
$$a \approx 1.226\ldots \text{ m/s}^2$$

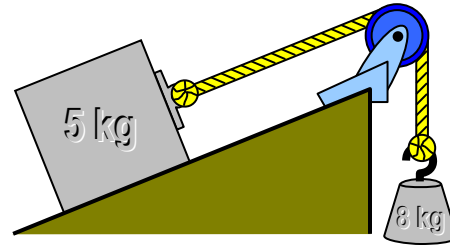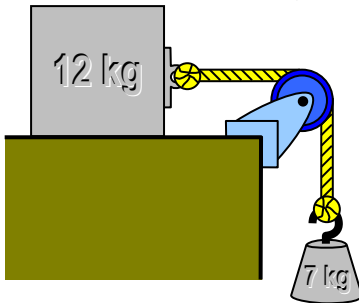$$F_T = 3(1.226) + 29.43 = 33.109\ldots \text{ N}$$

∴ The acceleration in the system is 1.2 m/s² and the tension is 33 N.

EXERCISES

A. For each of the weights in each diagram, determine acceleration, and for each rope in the diagram, determine tension. (All pulleys are massless. Use $g = 9.81 \frac{m}{s^2}$.)
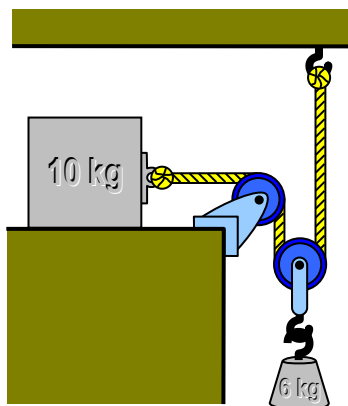
1) $\mu_k = 0.20$, hanging weight = 7.0 kg, block = 12.0 kg

2) $\mu_k = 0.15$, ramp = 20°, hanging weight = 8.0 kg, block = 5.0 kg

3) $\mu_k = 0.28$; hanging weights = 4.0 kg, 8.0 kg; block = 6.0 kg

4) $\mu_k = 0.420$, hanging weight = 35.0 kg, block = 15.0 kg, applied force = 350. N
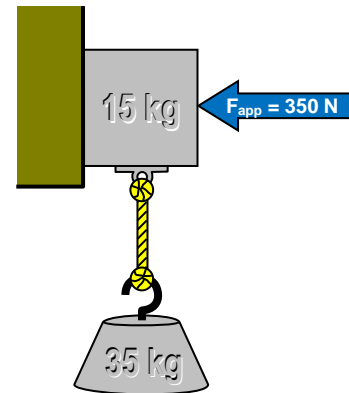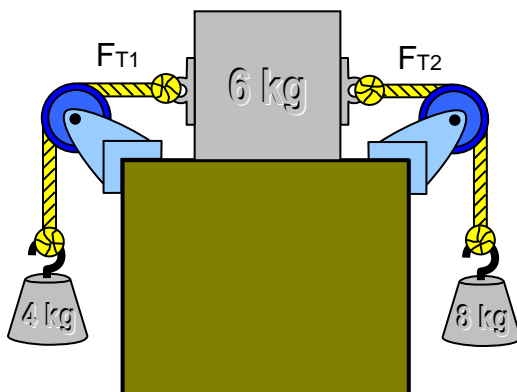
5) $\mu_k = 0.13$, hanging weight = 6.0 kg, block = 10.0 kg, $a_{10} \neq a_6$ [*Hint: For every metre that the 6-kg weight falls, how far does the block move?*]

SOLUTIONS

A: (1) $F_T = 52$ N, $a = 2.4 \frac{m}{s^2}$ (2) $F_T = 45$ N, $a = 4.2 \frac{m}{s^2}$ (3) $F_{T1} = 44$ N, $F_{T2} = 68$ N, $a = 1.3 \frac{m}{s^2}$ (4) $F_T = 103$ N, $a = 6.87 \frac{m}{s^2}$ (5) $F_T = 27$ N, $a_6 = 0.73 \frac{m}{s^2}$, $a_{10} = 1.5 \frac{m}{s^2}$