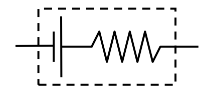
Batteries & Terminal Voltage

Every device that carries a current offers some resistance. That includes batteries — even though they may act as potential rises (increases in electrical potential) in a circuit, nothing is perfect, and the resistance of the material in the battery works against itself.

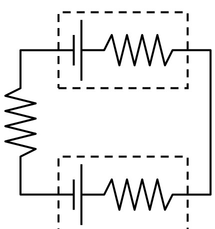
Remember that resistances serve as potential drops (decreases in electrical potential) in the direction of current flow always, even as part of a battery. The symbols that are usually used to represent a battery with an **internal resistance** are shown to the right. The dotted line groups the symbols together as one object.



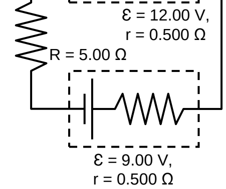
The **terminal voltage** of the battery is simply the sum of the potential differences from the two components in a "real" battery. The battery applies an electromotive force, or **emf** (symbol Ɛ, *not* E!), to the circuit that is expressed in volts, and the resistance adjusts that voltage *down* according to Ohm's Law: $V = IR$.

The equations for terminal voltage when the battery is discharging (providing power) and charging (taking power) are tricky. The difference is the direction the battery is facing on the circuit. If a battery is the lone power source on the circuit, it *must* be discharging. If there are multiple batteries and they're installed in both directions on the circuit, the higher voltage is discharging and the lower voltage is charging.

Look carefully at the circuit diagram on the right. The two batteries are pointing the same way, positive terminals on the right, negative terminals on the left… but it's not really true! From the perspective of the circuit, the positive terminals are directly connected to each other. They should be connected positive to negative, so one of the batteries is reversed on the circuit; the less powerful battery is getting energy from the more powerful one.



Let's say that the upper battery has the higher voltage and current is flowing clockwise. The upper battery has an emf and also act as a potential drop of its internal resistance times the current, so the cumulative voltage is $\Delta V = \mathcal{E} - Ir$. For the other battery, the one that's charging, the emf of the battery counts as a potential drop as well! The cumulative voltage is $\Delta V = -\mathcal{E} - Ir$. Both potential differences are negative, but we'd report the terminal voltage as positive, so it comes out as $\Delta V = \mathcal{E} + Ir$ — it looks like the internal resistance has become positive somehow, but no, the emf has become negative because the battery has been plugged in the other way around.



Example 1: Find the terminal voltages of each battery and the current.

Solution: We use the same procedure as for any circuit: find the equivalent resistance to determine current. The fact that some of the resistances are internal doesn't change anything. The resistances are all in series, so they add up: $R + r_{12} + r_9 = 6.00\ \Omega$. The Loop Law tells us:

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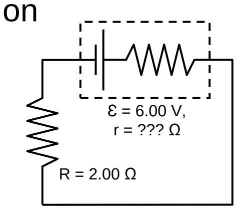
$$\Delta V_{circuit} = 12.00\text{ V} - 9.00\text{ V} - I(6.00\ \Omega) = 0 \Rightarrow I = 3.00\text{ V} \div 6.00\ \Omega = 0.500\text{ A}$$
$$\Delta V_{12} = 12.00 - 0.500 \cdot 0.500 = 11.75\text{ V}$$
$$\Delta V_{9} = 9.00 + 0.500 \cdot 0.500 = 9.25\text{ V}$$

Internal resistance detracts from the voltage across a battery powering a circuit, and increases the voltage across a battery acting as a load; that voltage is actually negative.

Example 2: A battery has a terminal voltage of 6.00 V when it's tested by itself, i.e., not as part of a circuit. Its terminals are connected by wires to a 2.00-Ω resistor and its voltage now reads 5.62 V. What is the internal resistance of the battery?

Solution: The terminal voltage of a battery away from a circuit (measured on an ideal voltmeter) is equal to its emf, so $\mathcal{E} = 6.00$ V. The circuit diagram appears at the right. The terminal voltage of the battery must be equal to the voltage across the resistor, since it's the only other thing on the circuit:



$$\Delta V_{res} = IR \ \Rightarrow\ 5.62\text{ V} = I(2.00) \ \Rightarrow\ I = 2.81\text{ A}$$

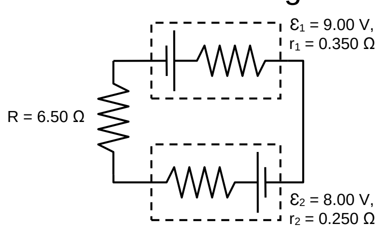
We can now calculate the internal resistance:

$$\Delta V = \mathcal{E} - Ir \ \Rightarrow\ 5.62\text{ V} = 6.00\text{ V} - (2.81\text{ A})r \ \Rightarrow$$
$$r = 0.38\text{ V} \div 2.81\text{ A} = 0.13523\ldots \approx 0.135\ \Omega$$

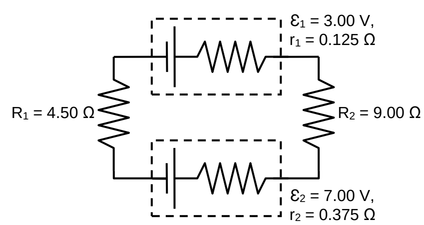
EXERCISES

A. Find the terminal voltages of each battery and the current.

1)



2)



B. A former physics instructor told his class about the time that a metal wrench was dropped in an auto mechanic's garage so that it landed on both terminals of a car battery. Let's do a back-of-the-envelope calculation to determine what happened.

First, we determine the resistance of the wrench. Looking online, we see that metal wrenches tend to be made of some kind of steel, a good conductor. To find the resistance of an object based on its composition, we use $R = \rho L / A$, where ρ (the Greek letter rho) is the resistivity of the material, L is the length of the resistor and A is the cross-sectional area of the resistor. Most metals have a resistivity on the order of 10^{-7} Ω·m, so let's use stainless steel at 6.9×10^{-7} Ω·m. (The other option I found was carbon steel, 1×10^{-10} Ω·m; it's worth keeping in mind that our answer could be $\sim 1/10{,}000$ what we get.)

The length of the resistor will be the distance between the posts of the car battery. I couldn't find that, but they tend to be arranged on the long dimension on the top of the battery. Car batteries come in various sizes, but most are between 8″ and 10″. Let's take the middle, 9″, convert to metric and round down to one sig fig. We'll use L = 20 cm, which certainly seems reasonable.

Likewise, looking at the dimensions of wrench handles, a wrench with a flat handle has a cross-sectional area of about 4 cm^2 = 4 × 10^{-4} m^2.

1) Given $R = \rho L / A$, $\rho = 6.9 \times 10^{-7}$ Ω·m, L = 20 cm and A = 4 × 10^{-4} m^2, determine the resistance of our theoretical wrench to one significant figure, in Ω.

We can also look up the voltage and internal resistance of a car battery. (The r for a car battery is *much* lower than, say, for AA batteries, which is why you can't use a handful of flashlight batteries to start your car.) Let's say that the car battery's emf is 12.8 V, and its internal resistance is 0.01 Ω, both typical values.

2) What was the current running through the wrench?

(You divided by resistance to get this answer, which means your answer could be ~10,000× higher, depending on the type of steel the wrench is made of.)

3) What was the battery's terminal voltage?

4) Determine the power output using $P = \Delta V \cdot I$. Because there's only one load resistor on the circuit, this is the power dissipated through the wrench as heat.

Now we want to know the effect of all that heat. Heat energy, Q, raises the temperature of an object based on that object's mass and the rate at which its material absorbs heat, called its specific heat capacity. Looking at the website that gave us the dimensions of the wrench, the mass comes out to about 800 g, and the specific heat capacity of steel is $c_{steel} = 420$ J/kg·°C.

5) Use the formulas $Q = mc\Delta T$ and $Q = P \cdot \Delta t$ to determine the rate at which the temperature of the wrench increases, $\Delta T/\Delta t$.

6) Given your answer to (5), what do you suppose happened to the wrench in the instructor's story?

SOLUTIONS

A: (1) $R_{eq} = 7.10$ Ω; $\Delta V_{circuit} = 17.00\text{ V} - 7.10I = 0$; $I = 2.39$ A; $\Delta V_1 = 8.16$ V; $\Delta V_2 = 7.40$ V

(2) $R_{eq} = 14.00$ Ω; $\Delta V_{circuit} = 4.00\text{ V} - 14.00I = 0$; $I = 0.286$ A; $\Delta V_1 = 3.04$ V; $\Delta V_2 = 6.89$ V

B: (1) $R \approx 3 \times 10^{-4}$ Ω (2) $I \approx 8 \times 10^3$ A (!!) (3) $\Delta V \approx 2.5$ V

(4) 2×10^5 W or 20 kJ/s (!!!!)

(5) $\Delta T/\Delta t = P/mc = (2 \times 10^5)/(0.8\text{ kg} \cdot 420\text{ J/kg·°C}) \approx 60$ °C/s

(6) The welding temperature of typical steel is between 900° C and 1100° C, which would be achieved in this process in under 20 s. The heat from resistance welded the wrench to the battery terminals. Add to this the fact that if someone had been foolish enough to try to salvage the situation by picking up the wrench, forget the heat — a current of 0.1 A can be fatal. The mechanic had to wait until the car battery went dead before it could be approached. Really, don't let this happen. It's incredibly dangerous.