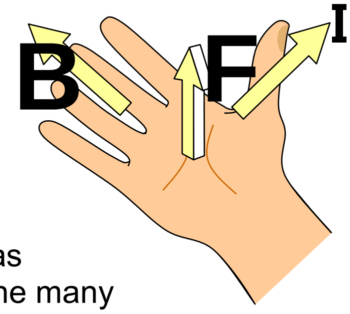
VANCOUVER COMMUNITY COLLEGE

Electromagnetism

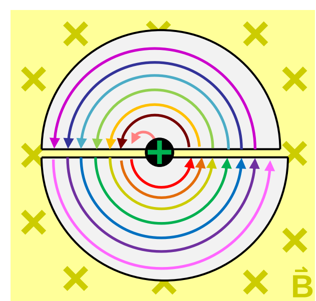
The Cyclotron



A **cyclotron** (also called a particle accelerator) is a device that uses the properties of charged particles in magnetic fields and potential differences to bring a particle up to a high speed. Recall that a charged particle moving in a magnetic field whose direction is perpendicular to the motion will experience a magnetic force perpendicular to both the motion and the field lines. The Right Hand Rule can help you figure out the directions: If the current (I, defined as movement of *positive* charges) is in the direction of the thumb, and the many magnetic field lines (B) are in the direction of the fingers, then the particles will be pushed in the direction of the palm by a magnetic force (F). In a constant magnetic field, the particle will be deflected in a new direction, the direction of force will change, and the particle will end up moving in a circle, in a process called **cyclotron motion**. The formula for the radius of that motion is shown.

$$r_{cyc} = \frac{mv}{qB_\perp}$$

The cyclotron itself consists of two semicircular plates of metal called "**dees**" (since the letter D is more or less a semicircle) separated by a small gap, immersed in a magnetic field orthogonal to the planes of the plates. A charged particle, such as a proton, starts in the centre and a potential difference in the gap between the dees propels it towards one of them. That same potential difference won't exist within a piece of metal, so the particle comes under the influence of the magnetic field only, and travels in a semicircle back towards the gap. By this point the potential difference has been reversed, propelling the particle to the other dee. Each time the particle crosses the gap, the electric field between the dees does work on the particle, increasing its kinetic energy, and therefore its velocity. The increase in the particle's velocity also increases the radius of its motion inside the dees. Each trip through a dee takes the same amount of time — the greater distance is offset by the higher speed. The frequency at which the particle crosses the dees is the **cyclotron frequency**; its formula is shown.



$$f_{cyc} = \frac{qB}{2\pi m}$$

Example 1: A cyclotron is accelerating an alpha particle (He^{2+}, $m_\alpha = 6.65 \times 10^{-27}$ kg) in the presence of a magnetic field of 1.6 T. The alternating voltage across the dees is 30. kV, and the entire process takes 20.37 μs. What is the final radius of the alpha particle?

Solution: This is a long problem. We know everything to calculate the cyclotron radius except the velocity. We can calculate the velocity from the kinetic energy of the particle, which we can determine from the work done on the particle in each crossing (which we can know now) and the number of crossings. Finally we can get the number of crossings with the cyclotron frequency and the total time. We start there.

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The frequency of the cyclotron is the number of circular trips per second. Each trip, no matter what the alpha particle's current radius is, take the same amount of time, so the frequency is constant.

$$f_{cyc} = \frac{qB}{2\pi m} = \frac{(2 \cdot 1.602 \times 10^{-19}\ \text{C})(1.6\ \text{T})}{2\pi(6.65 \times 10^{-27}\ \text{kg})} = 1.2269\ldots \times 10^{7}\ \text{Hz}$$

$$n_{cycles} = f_{cyc} \cdot t_{ttl} = (1.2269 \times 10^{7}\ \text{Hz})(20.37 \times 10^{-6}\ \text{s}) = 249.92\ldots \approx 250\ \text{cycles}$$

Each cycle is two crossings. $\therefore\ n_{crossings} = 500$

In each crossing, work is done on the particle to increase its kinetic energy. Since we know how many crossings, we can calculate the total final kinetic energy of the alpha particle, which was accelerated from rest:

$$W = q \cdot \Delta V = (2 \cdot 1.602 \times 10^{-19}\ \text{C})(30 \times 10^{3}\ \text{V}) = 9.612 \times 10^{-15}\ \text{J}$$

$$K_f = n_{crossings} \cdot W = 500(9.612 \times 10^{-15}\ \text{J}) = 4.806 \times 10^{-12}\ \text{J}$$

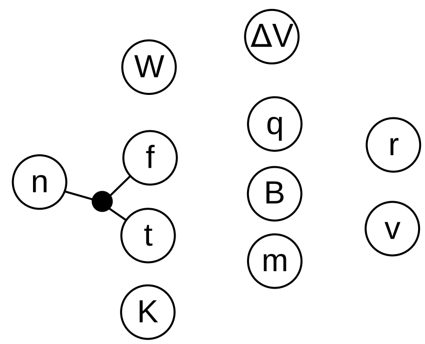
$$K_f = \tfrac{1}{2}mv^2 \Rightarrow v = \sqrt{\frac{2K_f}{m_\alpha}} = \sqrt{\frac{2(4.806 \times 10^{-12}\ \text{J})}{6.65 \times 10^{-27}\ \text{kg}}} = 3.8019\ldots \times 10^{7}\ \text{m/s}$$

We now have the final velocity of the particle, and at last we can finish the calculation for the radius:

$$r_{cyc} = \frac{mv}{qB} = \frac{(6.65 \times 10^{-27}\ \text{kg})(3.8019 \times 10^{7}\ \text{m/s})}{(2 \cdot 1.602 \times 10^{-19}\ \text{C})(1.6\ \text{T})} = 0.49318\ldots \approx 49\ \text{cm}$$

EXERCISES

A. The solution to Example 1 used six different formulas: cyclotron frequency, cyclotron radius, electrical work, kinetic energy, number of crossings, and frequency as cycles per second. Complete the diagram at right by creating a black dot representing each formula, then drawing lines from each dot to the variables used in that formula. The frequency formula has been done as an example.



B. When you know all the variables in a formula except one, you can solve for the last variable. Consider the following problem: "A cyclotron is used to accelerate a proton ($q = e$, $m_p = 1.602 \times 10^{-19}$ kg). A voltage of 14.0 kV is applied to the dees alternating at 1.81×10^{7} Hz until the proton travels a path of radius 27.3 cm. How many microseconds does this process take?"

1) Shade the circles containing known quantities from the problem in the diagram from A. What two variables can you solve for immediately? Shade over the dots for the equations you would use and the lines to those two quantities. Finally shade those quantities.

2) You can now solve for one more quantity. Continue this process to solve the problem.

SOLUTIONS

A: Diagrams will vary.

B: (1) $B = 1.1869\ldots$ T; $W = 2.2428 \times 10^{-15}$ J

(2) $v = 3.1047\ldots \times 10^{7}$ m/s; $K = 8.0584\ldots \times 10^{-13}$ J; $n_{cycles} = 180$ (179.65…);
$t = 9.9448\ldots\ \mu\text{s} \approx 9.94\ \mu\text{s}$