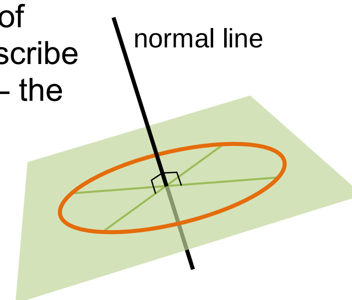
Induced Current & Lenz's Law

When we pass a magnet through a simple coil of wire (with no power source needed), a small current spontaneously flows in the wire. To be able to measure how much current will flow, we need to quantify changes in magnetic fields.

We'll simplify the problem, first: we'll only look at flat loops of wire. Then we need some way of turning the plane (the flat surface) that the loop sits in into a direction. We could choose a representative vector that lies in the plane, but there are lots of those, and no clear way to choose just one. The clever way that we describe the "direction" of a plane is to talk about the **normal line** to the plane — the line that is perpendicular to the plane in every way.



Now we can describe how much of a magnetic field is passing though a loop. This depends on the area of the loop (A), the magnetic field's strength (B), and the degree to which the field lines pass directly through the loop. The closer the field lines come to being parallel to the normal line through the loop, the greater the effect on the current. We call this measurement of magnetic fields through a planar area **magnetic flux**, measured in Webers ("*vay*-bers", Wb):

$$\Phi = BA\cos\theta,$$

where θ is the acute angle between the field lines and the normal line

Magnetic flux describes the penetration of a magnetic field through a loop; current develops when the amount of penetration changes, whether it increases or decreases. The faster it changes, the greater the effect. If we have multiple coils of wire in the same place (as opposed to a spring-shaped solenoid, where the turns of wire are separated by some distance in the direction of the normal line), each coil magnifies the effect:

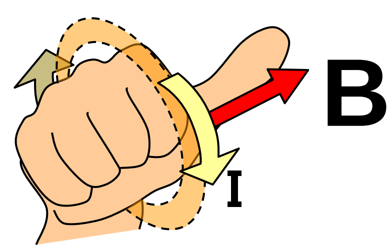
$$\text{Faraday's Law: } \mathcal{E} = -N\frac{\Delta\Phi}{\Delta t},$$

where $\mathcal{E}$ is induced emf (V), N is the number of coils, and Δt is in (s)

The formula has a minus sign in it, which is meant to indicate the direction of flow of current. We also have **Lenz's Law** to help us decide the direction of current in the loop:

Current will flow in the loop so as to oppose the change in the magnetic flux, i.e., to increase it if it decreased, and decrease it if increased.

Recall that current loops create their own magnetic fields according to the Right-Hand Rule: When current flows in the direction of the curled fingers, the magnetic field points in the direction of the thumb.



The induced current's own magnetic field will increase flux when it points in the same direction as the ambient magnetic field, and it will decrease flux when it opposes the ambient magnetic field.

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Example 1: A single circular loop of wire lies in the plane of this worksheet. Its radius is 10. cm. It is surrounded by a magnetic field pointing out of the page with a strength of 0.050 T. In the course of one minute, the field decreases to 0.020 T. What is the total induced emf in the loop, and will current flow clockwise or counterclockwise in the loop?

Solution: The number of coils of wire is N = 1, and the time difference Δt = 60 s. We need to know the change in magnetic flux. Since the normal line to the loop is parallel to the field lines, magnetic flux is at full strength.

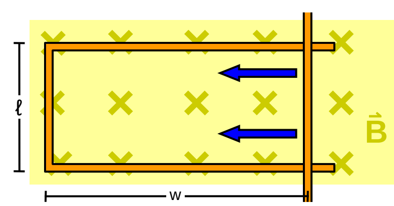
$$\Phi_i = (0.050\text{ T})[\pi \cdot (0.10\text{ m})^2](\cos 0°) = 0.0015708\ldots\text{ Wb}$$
$$\Phi_f = (0.020\text{ T})[\pi \cdot (0.10\text{ m})^2](\cos 0°) = 0.00062832\ldots\text{ Wb}$$
$$\Delta\Phi = 0.00094248\ldots\text{ Wb}$$

$$|\mathcal{E}| = N\frac{\Delta\Phi}{\Delta t} = 1 \cdot \frac{9.4248 \times 10^{-4}\text{ Wb}}{60\text{ s}} = 15.708\ldots\ \mu\text{V} \approx 16\ \mu\text{V}$$

It's much easier to get the direction of current flow from Lenz's Law. The ambient magnetic field lines (from the 0.050-T field) point out of the page, but they're getting weaker. Lenz's Law says that the coil's field points in the same direction as the ambient field (adding a tiny bit back to the field strength, and opposing the decrease). The current in the coil makes a magnetic field out of the page, so our Right Hand tells us the current flows counterclockwise.

MOVING WIRES

Electromotive force is created by a change in magnetic flux, ΔΦ = Δ(BA). Changing the magnetic field strength can accomplish this, (ΔB)A, but so can changing area, B(ΔA). Consider a rectangular loop made up of one U-shaped wire and a straight wire in contact with both ends of the U. For simplicity's sake, put the loop in a magnetic field parallel to the normal line to the loop. If we slide the straight wire towards the "bottom" of the U, there's less penetration through the loop simply because the loop is getting smaller. If the wire is moving with constant velocity v, then we can calculate the magnitude of emf as:



$$|\mathcal{E}| = N\frac{\Delta\Phi}{\Delta t} = N\frac{B(\Delta A)}{\Delta t} = NB\frac{\ell(\Delta w)}{\Delta t} = NB\ell\frac{\Delta w}{\Delta t}$$

For problems like this, N is invariably equal to 1 — it's hard enough maintaining the electrical contact for this situation, let alone several loops. The rate at which the width of the loop is changing is the velocity of the wire, v, so:

$$|\mathcal{E}| = B\ell v$$

This formula only works if all of these things are mutually perpendicular. If not, we have to use sine or cosine of the angles involved to take the components of $\vec{B}$ and $\vec{v}$ that are normal to ℓ.

Example 2: If the magnetic field in the diagram has a strength of 0.05 T, the bottom of the U is replaced by a 15-Ω resistor which creates a rectangular loop 12 cm long, and the wire is moved steadily at 0.8 $^{cm}/_{s}$ away from the resistor, how much current is generated, and which way does it flow?

Solution: Electromotive force is easy to calculate with the new formula:

$$\mathcal{E} = (0.05\ \text{T})(0.12\ \text{m})(0.008\ ^{m}/_{s}) = 48\ \mu\text{V}.$$

The current flowing through the resistor can be found with Ohm's Law:

$$I = V/R = 48 \times 10^{-6}\ \text{V} \div 15\ \Omega = 3.2\ \mu\text{A} \approx 3\ \mu\text{A}.$$

The direction of current is found using Lenz's Law. The magnetic field lines point into the page. The loop is expanding, so flux is increasing in that direction. The current will be generated so as to oppose that flux — magnetic field lines will point in the opposite direction, out of the page. The Right-Hand Rule says current will flow counterclockwise.

The interesting thing is that the emf is developed in a moving conductor even if there's no loop to speak of — current doesn't flow, since current does need a loop — but a potential difference is developed across the length of the conductor.

Example 3: A metal rod, 1.0 m long and oriented north-south, is thrown eastward through a constant upward-pointing magnetic field of 0.42 T. The rod is thrown with an initial speed of 14 $^{m}/_{s}$ at 62° above the horizontal. What is the potential difference across the rod during its freefall?

Solution: We recall from more traditional projectile motion problems that we can split the problem into two parts: a vertical component, affected by gravity, and a horizontal component which does not have acceleration in it. The vertical part of this problem has no bearing on the potential difference — movement in the same direction as the magnetic field lines doesn't "cut" any field lines, so movement up or down does not create an emf.

The forward/horizontal motion of the rod does, however, since the rod, its motion, and the field lines are mutually perpendicular. The conditions are met for a motional emf, and we just need to calculate the eastern component of the throw.

$$v_{east} = 14\ ^{m}/_{s} \cos 62° = 6.5726\ldots\ ^{m}/_{s}$$
$$\mathcal{E} = (0.42\ \text{T})(1.0\ \text{m})(6.5726\ ^{m}/_{s}) = 2.7605\ldots \approx 2.8\ \text{V}$$

EXERCISES

A. 1) Example 3 demonstrated that a metal rod, not part of a loop, thrown through a magnetic field will have an emf while it's in motion. Why wouldn't a square metal loop, 1.0 m × 1.0 m, have a current flowing in it if it's thrown flat (always horizontal, no spinning) inside the same field under the same conditions? Use the concept of flux to give one reason, and use the emf from the example to give another.

2) Instead of throwing the loop across the field, you hold the loop horizontal within the field with both hands on one side of the loop. You release your grip slightly, and the loop drops so it hangs straight down from your hands. Did any current flow in the loop? Explain.

B. Calculate the emf in the conductors in the situations described. If current would flow, determine which way.

1) A metal coat hanger ($A = 23\ cm^2$) sits flat on top of a speaker that is generating a 0.02-T magnetic field directly upwards through its top. The speaker powers down in 0.036 s, and the field dissipates.

2) A screen door 2.10 m tall squeaks open at 4.9 m⁄s, maintaining metal-on-metal contact, travelling 0.96 m. The wall that the screen door is in stalled in is perfectly east-west in Vancouver, where $\vec{B}_{earth} = (17.6\hat{\imath} + 5.04\hat{\jmath} + 50.8\hat{k})\ \mu T$ [(x, y, z) = (north, east, up)]

3) A boy scout holds out a compass perfectly flat while he rides in an elevator here in town. The compass needle, which is metallic and 3.00 cm long, aligns itself with the local magnetic field, parallel to $17.6\hat{\imath} + 5.04\hat{\jmath}$. The elevator is ascending at 8.0 m⁄s. (See the previous question for the local geomagnetic field.)

4) A car with a vertical radio antenna 75 cm long is being driven to North Vancouver across the Lions Gate Bridge at 50. km⁄h on a bearing of N 26.6° E. Assume the car experiences no change in altitude while this happens. (Again, see question (2) for the local geomagnetic field.)

C. A rare earth magnet dropped through a copper pipe will fall more slowly than gravity would dictate. Describe why this would be true using the Law of Conservation of Energy.

SOLUTIONS

A: (1) The area of the loop exposed to the field and the field strength stay constant, so the magnetic flux through the loop never changes. Current will not flow in the loop.

For the emf, the whole loop will be polarized and the south end of both north-south sides of the loop will be at 2.8 V higher potential than the north end, but in a current loop on one side the south end is at higher potential and on the opposite side the north end has to be at higher potential. (Draw a simple circuit to prove this to yourself.) There's no reason for charge to flow all the way around the loop.

(2) Current will flow while the loop is moving since the area exposed to the magnetic field changes from $1.0\ m^2$ to zero.

B: (1) 1.3 mV, ccw viewed from above (2) 18 mV, cw when viewed from the north
(3) 12.1 μV, no current (4) $\Delta\theta \approx 10.6°$ so multiply by sin 10.6°, 460 μV, no current

C: The pipe would act like a conducting loop, and the moving magnetic field creates a potential difference in the pipe; charges in the metal of the pipe will move. Work is being done on the moving charges ($W = q\Delta V$), and the source of energy for that work is the gravitational potential energy the magnet had at the top of the pipe, and so less of that energy gets converted to kinetic energy, and the magnet falls with a lower velocity.