



Tips and Tricks for Normal Distributions

There are two types of problems you will encounter with the normal distribution and sampling distributions. The first type of problem asks how often a particular result occurs. More specifically, we want to know how often more than a particular result occurs or how often less than a particular result occurs. The second type of problem asks for the measurement (x) that results in a desired or stated probability level.

How do you know it's a type 1 problem?

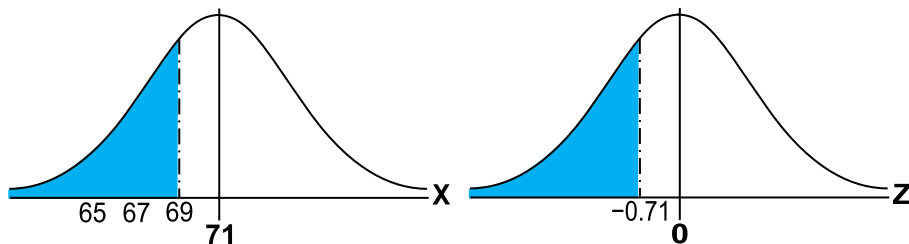
You are given: the mean of the population (μ), a measurement from the population (x) or the mean of a sample of size n (\bar{x}) and the standard deviation of the population (σ).
Tip: if you're not sure if you've been given a measurement or probability, check the units with the number. If the units match the units of the mean, it's a measurement!

To solve: Calculate z and then look up the probability. If the area of the distribution we're interested in is to the left of our measurement, we use the probability from the table. If the area of the distribution we're interested in is to the right of our measurement, we take 1 minus the probability from the table.

But how do we know which area we're concerned about? Here are the most common phrases that tell us which area or probability (left or right) the problem is asking about.

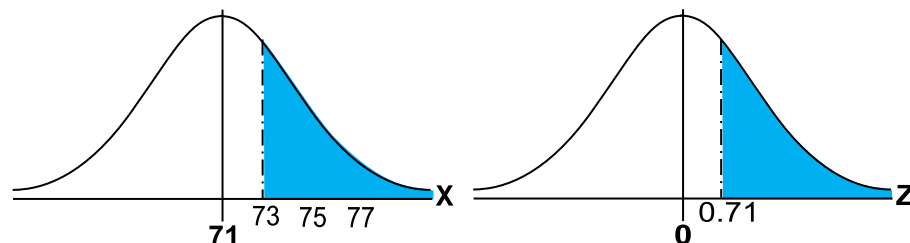
Phrases indicating the area to the left of a measurement: (note this is just the AREA, the measurement itself could be to the left or right of the mean)

"fewer than"
 "less than"
 "below"
 "no more than"
 "maximum"



Phrases indicating area to the right of a measurement: (note this is just the AREA, the measurement itself could be to the left or right of the mean)

"more than"
 "higher than"
 "above"
 "at least"
 "exceed"
 "minimum"



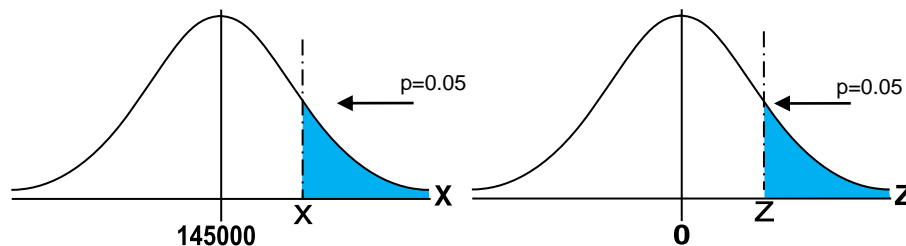
How do you know it's a type 2 problem?

You are given: the desired probability, usually as a percentage (p), population mean (μ) and the standard deviation of the population (σ).

Tip: It's a probability if it mentions: percentage (%), proportion, probability, or p .

Steps to solve: First decide if it's a left or right area of interest. The same clue phrases above still apply. One extra clue for questions indicating a right tail would be "the top ___% of the population..." One extra clue for questions indicating a left area would be questions guaranteeing a lifetime of a product and wanting to only replace ___%. Then find the given probability or 1 minus the given probability in the z-score table. Look up the z-score that goes with the probability. Use z to solve for x ($x = z\sigma + \mu$). Occasionally a measurement (x) will be given and you are asked to find the population mean. The process is the same for steps one and two, but in step three we solve for μ instead.

Example: The average oil production per day in barrels is 145,000 with a standard deviation of 315 barrels. In the top 5% of production days, more than how many barrels of oil per day will be produced?



Answer: Since the probability indicates the TOP 5% of days, and we see "more than" we know it is a right-tail test. In order to find, x , the number of barrels, we first need to find the probability to the left of $x = 1 - 0.05 = 0.95$. Look up 0.95 in the probability space of the z-score table. This corresponds to a z-score of 1.65. Now plug into the formula for $x = (1.65) * 315 + 145,000 = 145,519.75$ barrels or 145,520 barrels.

Common Sense Check:

Drawing the distributions described in stats problems is extremely helpful for determining if you've come up with the right answer. If you've drawn a diagram that shows the area to the right is of interest, and the measurement value is to the RIGHT of the mean, you would expect a probability of less than 0.50. Similarly if you were given a probability of 10% and you knew it referred to the right tail, you would expect to be finding a measurement value that is greater than the mean.

